KKTとLagrange（ボツ案）

交通研究室　学部４年
日下部　達哉

1. KKTの概要

- クーンタッカー条件
  …不等式による制約がある中で、
  ある関数を最大／最小化する

- ラグランジュの未定乗数法
  …等式による制約がある中で、
  ある関数を最大／最小化する

1. KKTの概要

<u>問題</u>

$$min.\ \ f(\boldsymbol{x})$$

$$subject\ to\ \ \ g_j(\boldsymbol{x}) \geq b_j$$

（最適解を出す座標を $\boldsymbol{x} = \boldsymbol{x}^*$ とする）

1. KKTの概要

必要条件

$$① \quad \frac{\partial f(\boldsymbol{x}^*)}{\partial x_i} = \sum_j u_j * \frac{\partial g_j(\boldsymbol{x}^*)}{\partial x_i}$$

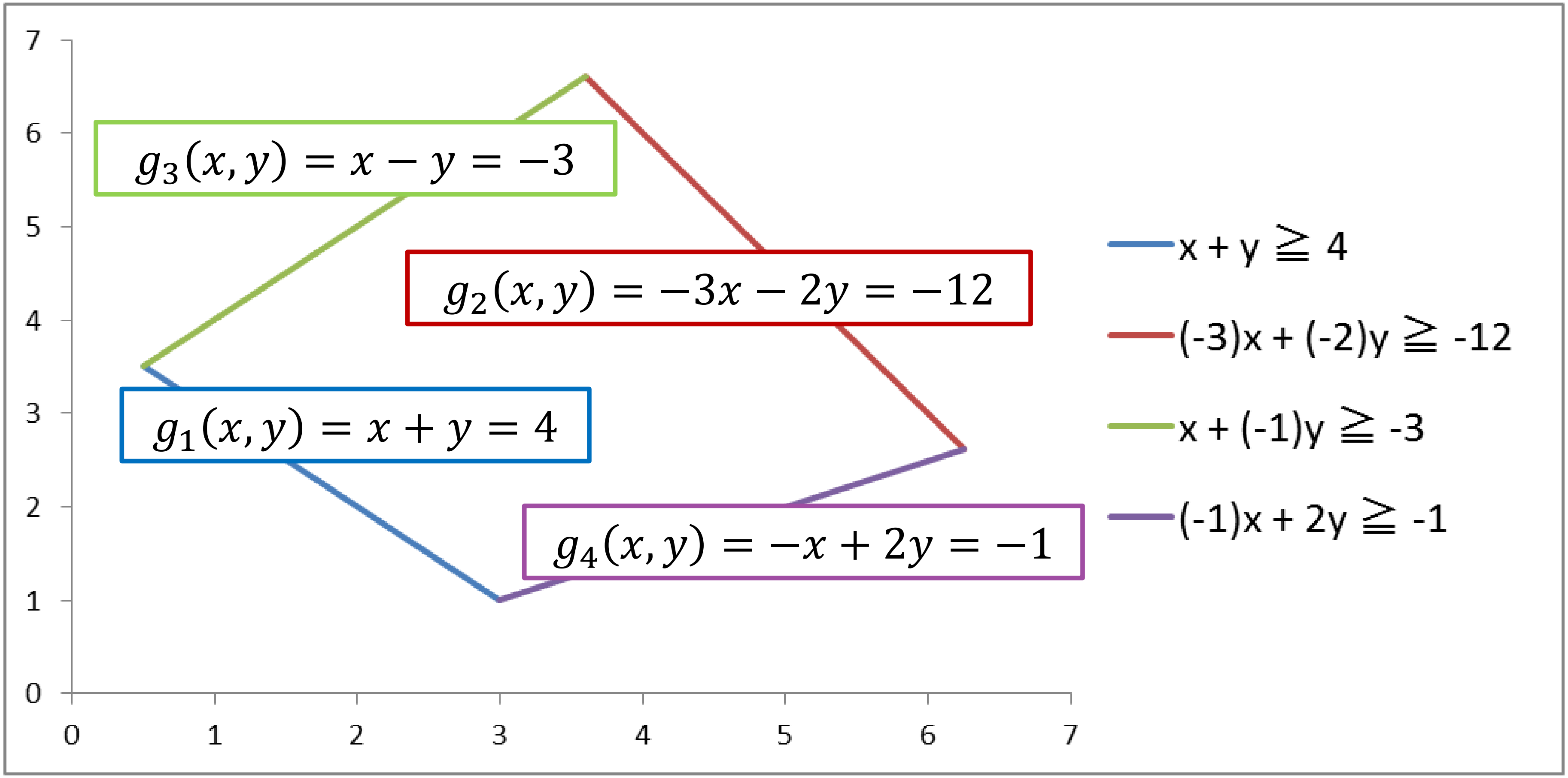
$$② \quad u_j \geq 0$$

$$③ \quad u_j * \left(b_j - g_j(\boldsymbol{x}^*)\right) = 0$$

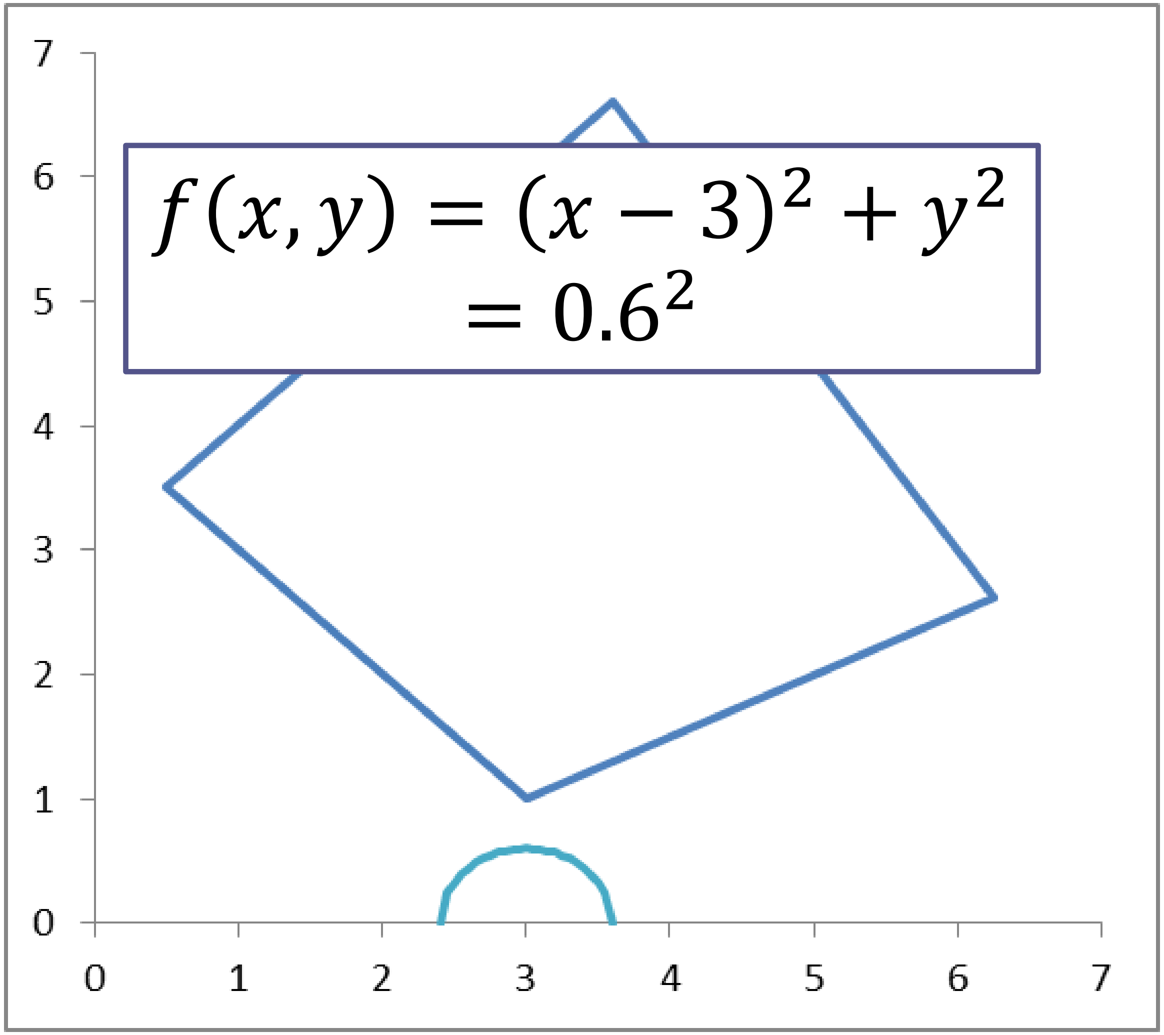
$$④ \quad b_j - g_j(\boldsymbol{x}^*) \geq 0$$

2. KKTの例題

- 一番目の例...きちんと最小値を求めている
- 二番目の例...きちんと最小値を求めていない

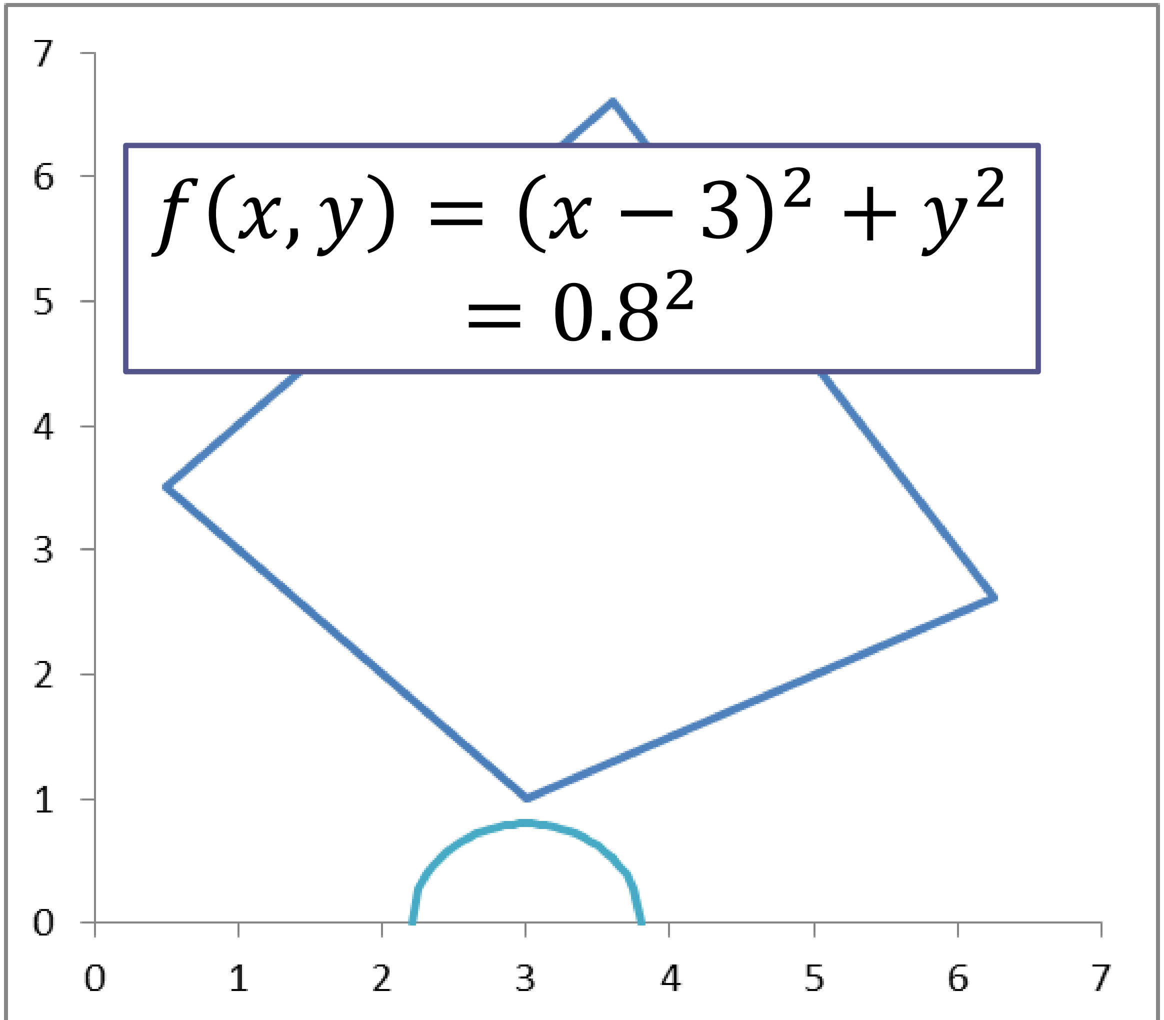


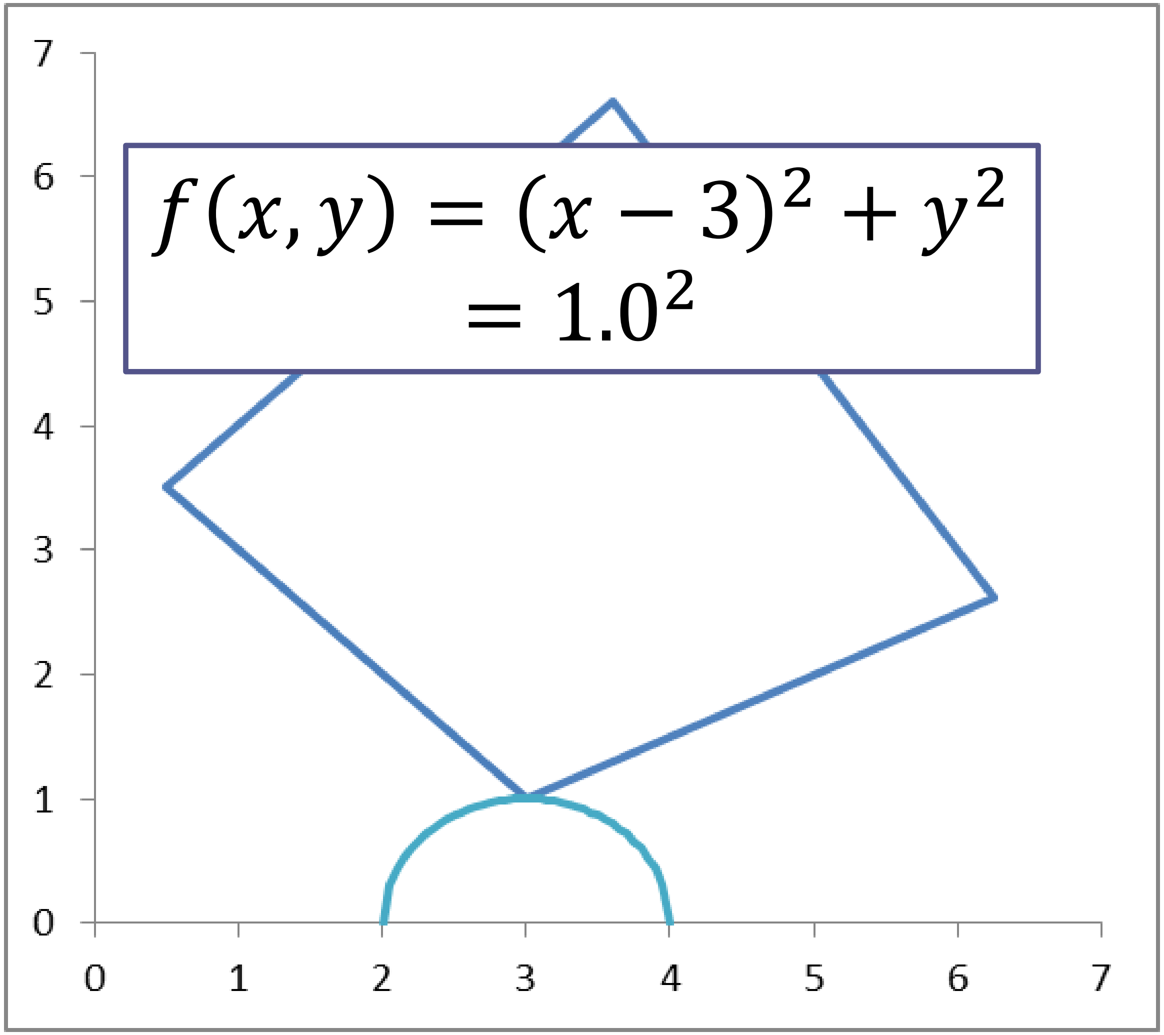
$$min.\ f(x,y) = (x-3)^2 + y^2$$



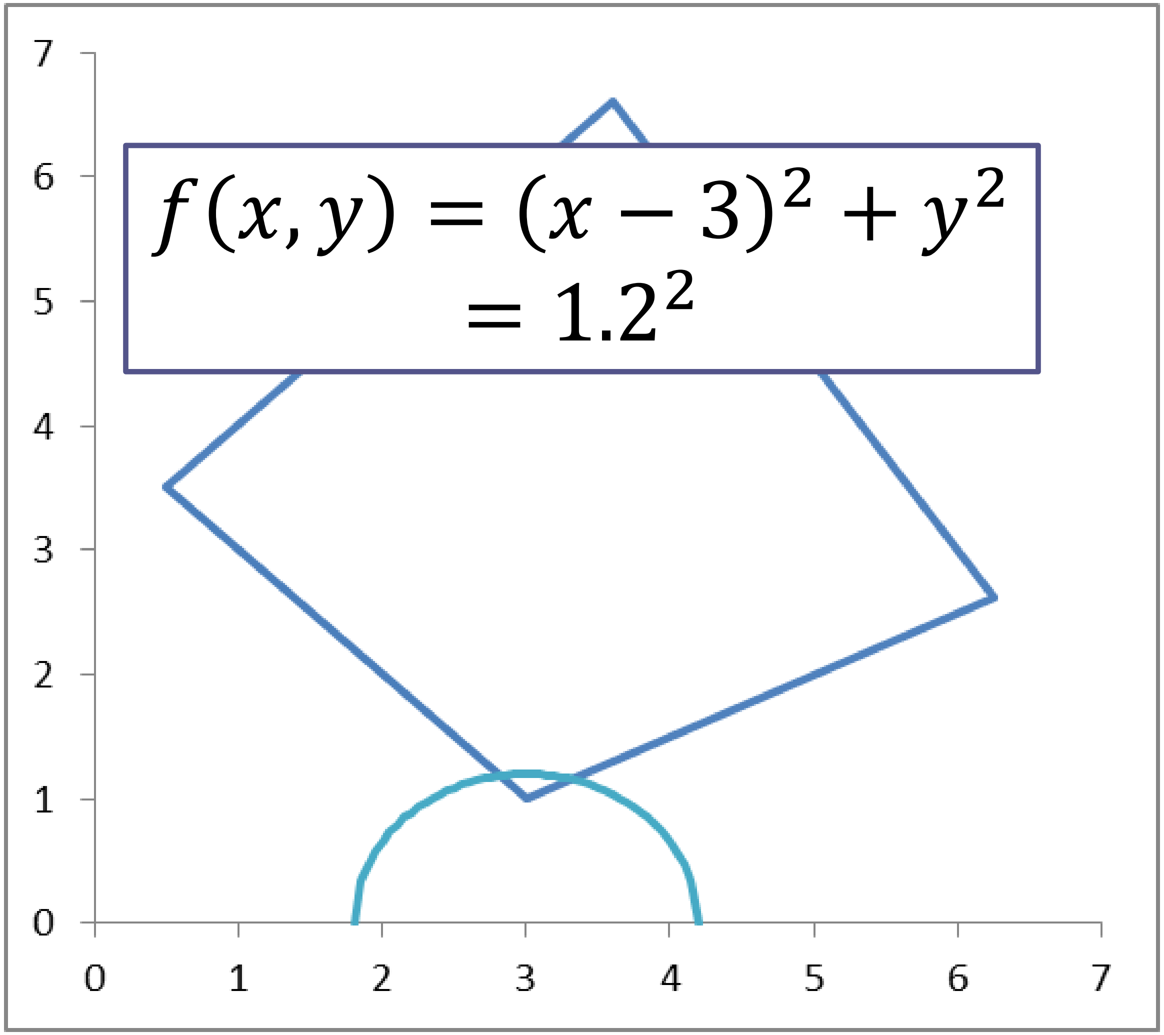
$$f(x,y) = (x-3)^2 + y^2 = 0.6^2$$

$$f(x,y) = (x-3)^2 + y^2 = 0.8^2$$

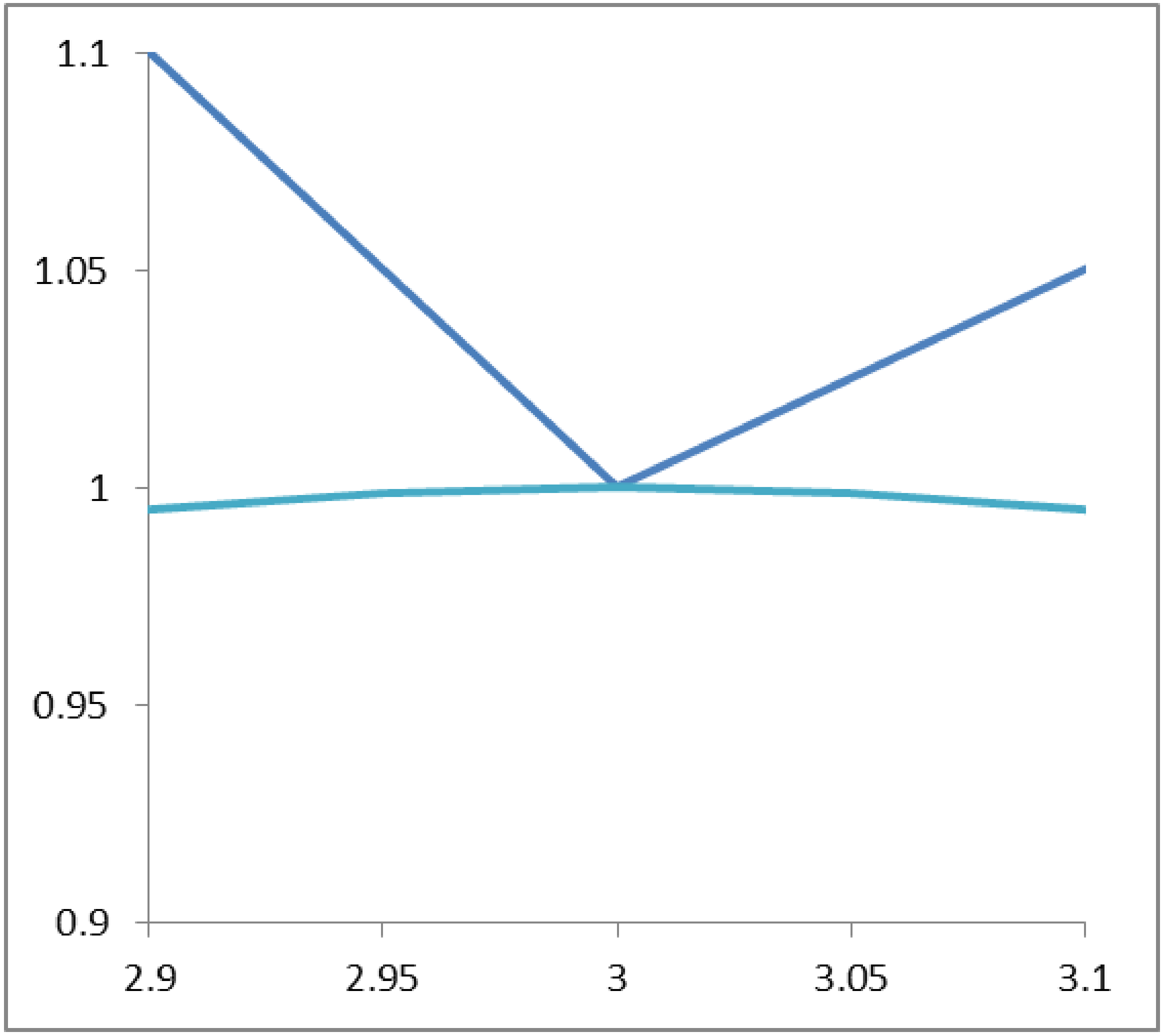




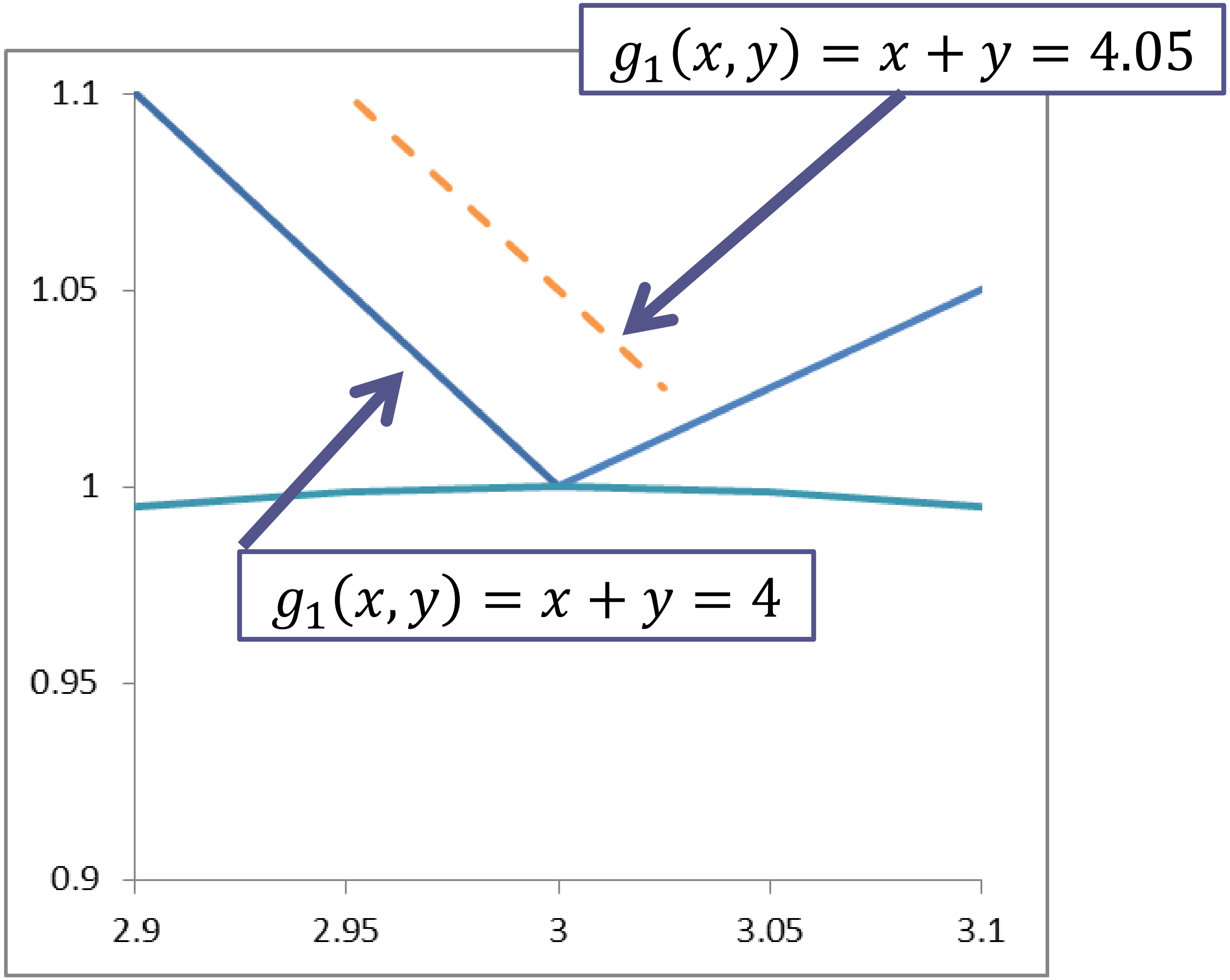
$f(x,y) = (x-3)^2 + y^2 = 1.0^2$
0
1
2
3
4
5
6
7

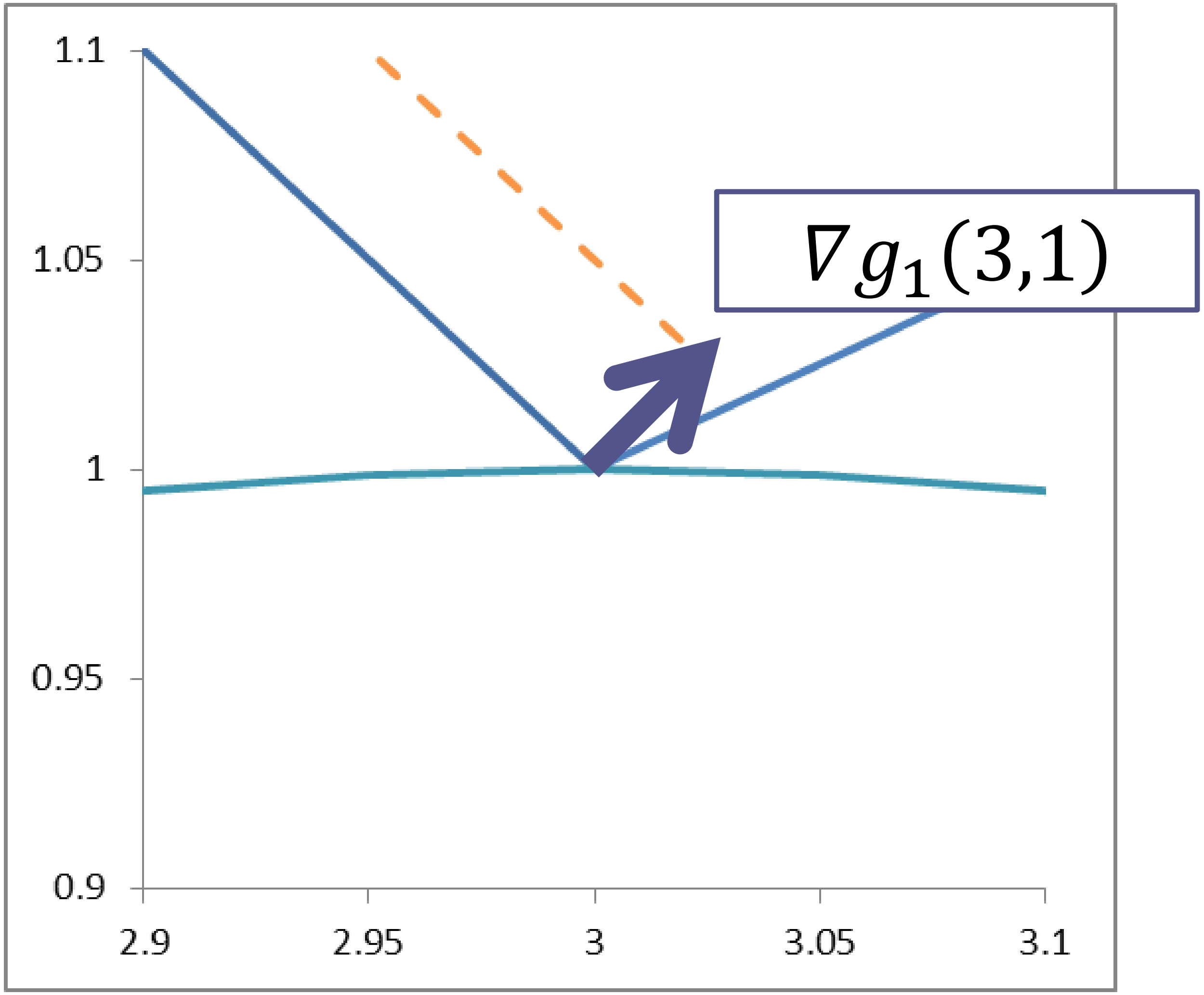


$$f(x,y) = (x-3)^2 + y^2 = 1.2^2$$

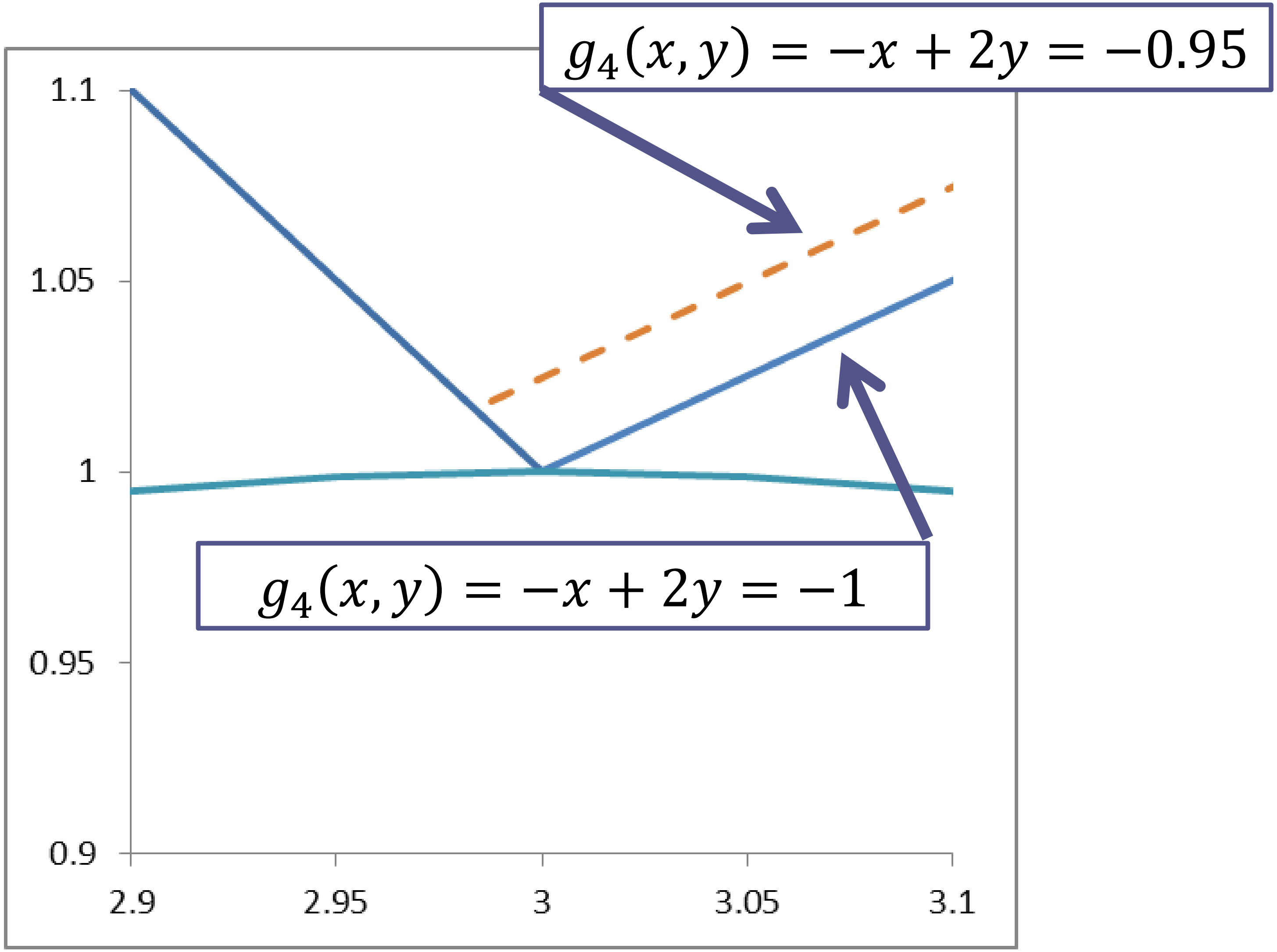


1.1
1.05
1
0.95
0.9
2.9
2.95
3
3.05
3.1

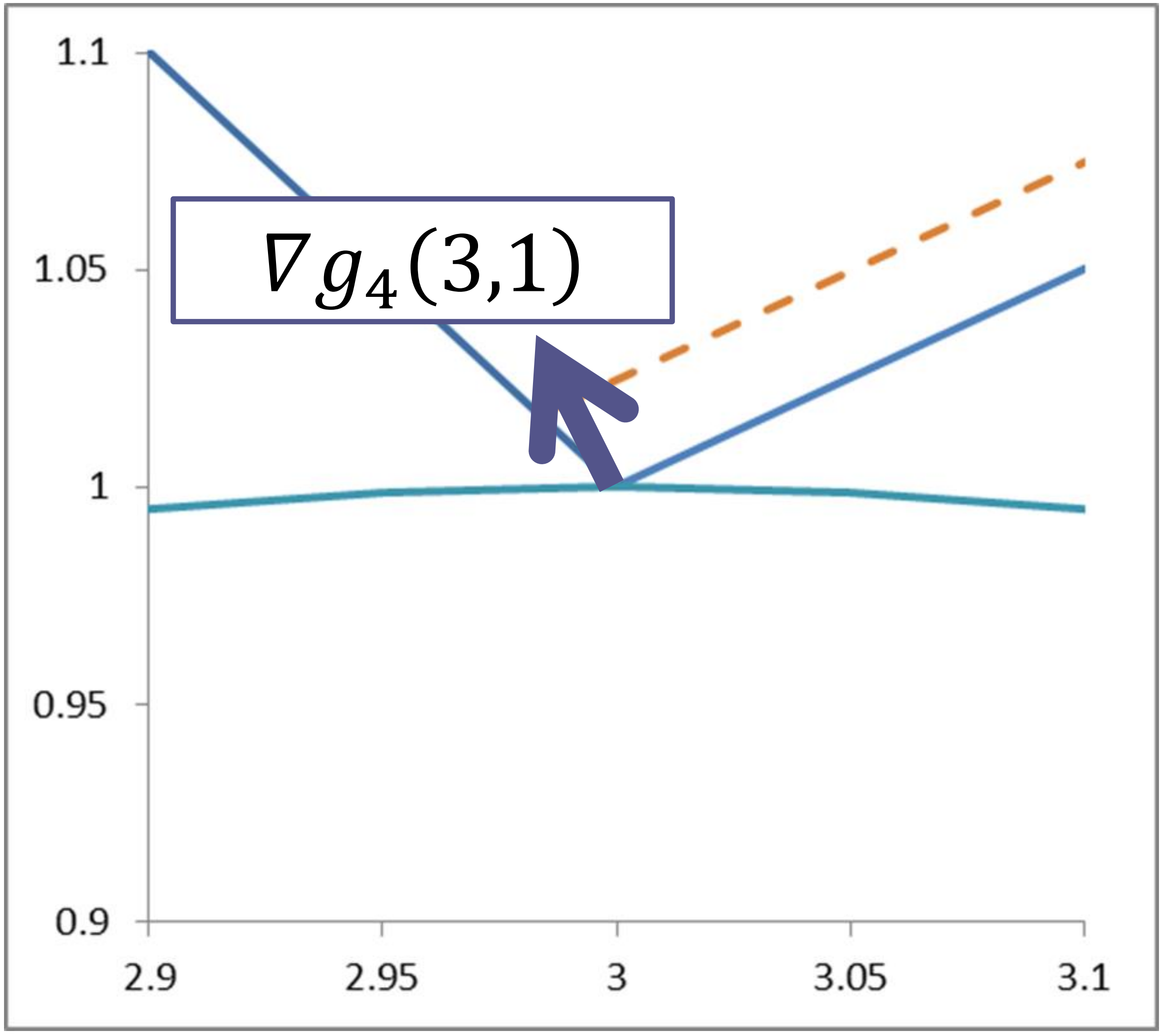

$g_1(x,y) = x + y = 4.05$
$g_1(x,y) = x + y = 4$
1.1
1.05
1
0.95
0.9
2.9
2.95
3
3.05
3.1



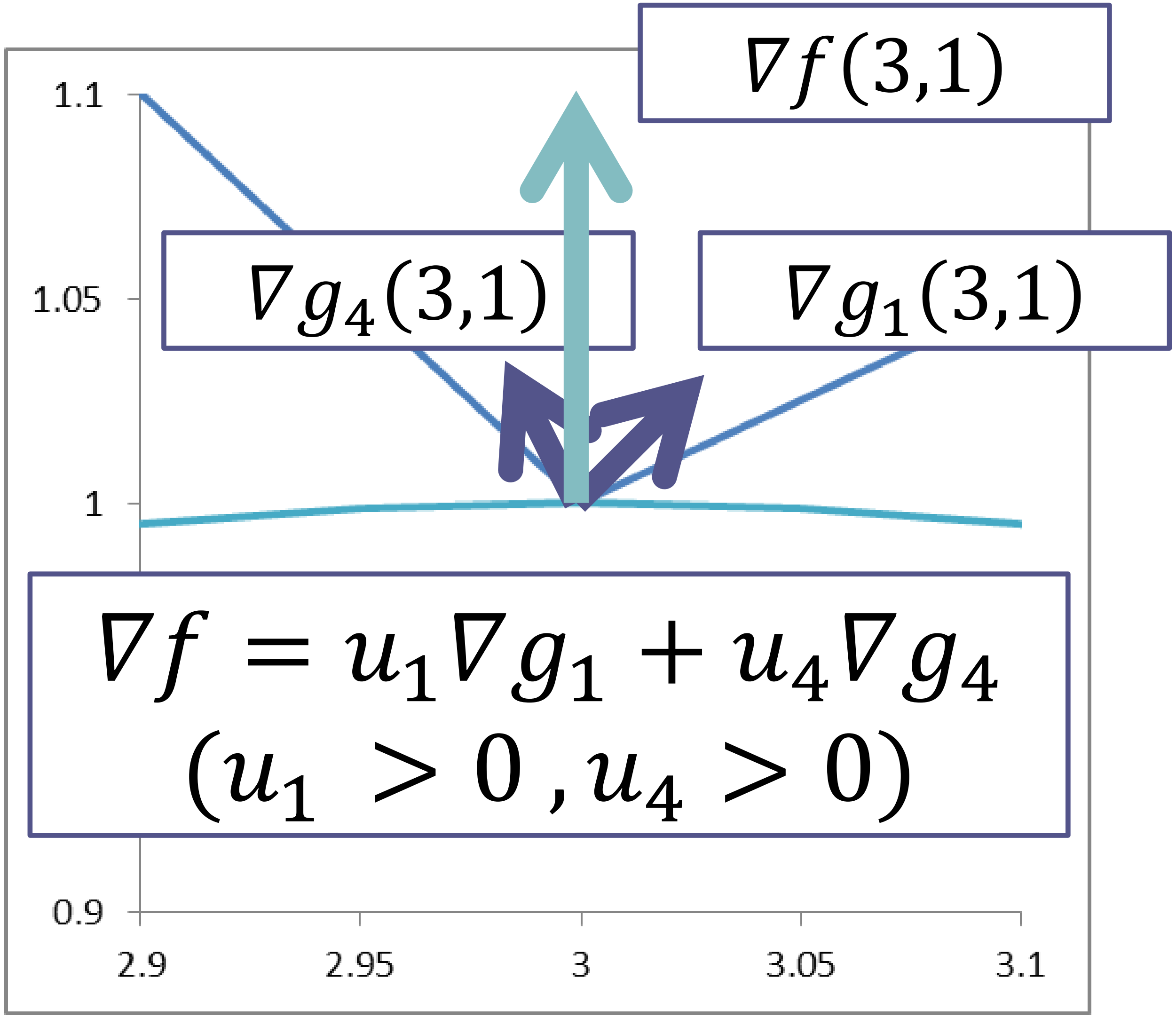
1.1
1.05
1
0.95
0.9
2.9
2.95
3
3.05
3.1
$\nabla g_1(3,1)$



$g_4(x,y) = -x + 2y = -0.95$
$g_4(x,y) = -x + 2y = -1$
1.1
1.05
1
0.95
0.9
2.9
2.95
3
3.05
3.1



1.1
1.05
1
0.95
0.9
2.9
2.95
3
3.05
3.1
$\nabla g_4(3,1)$


$\nabla f(3,1)$
$\nabla g_4(3,1)$
$\nabla g_1(3,1)$
$\nabla f = u_1 \nabla g_1 + u_4 \nabla g_4$
$(u_1 > 0, u_4 > 0)$
1.1
1.05
1
0.9
2.9
2.95
3
3.05
3.1

g_1とg_4の境界線上でfが最小化される

$\ldots u_1 > 0\,,\, u_4 > 0$

$\ldots g_1(3,1) = 4\,,\, g_4(3,1) = -1$

g_2とg_3は何の影響も与えない

$\ldots u_2 = 0\,,\, u_3 = 0$

$\ldots g_2(3,1) \neq 4\,,\, g_3(3,1) \neq -1$

② $u_j \geq 0$　③ $u_j * \left(g_j(\boldsymbol{x}^*) - b_j\right) = 0$

前頁から $\nabla f(\boldsymbol{x}^*) = \sum_j u_j * \nabla g_j(\boldsymbol{x}^*)$

一般に $\nabla f(\boldsymbol{x}) = (\frac{\partial f(\boldsymbol{x})}{\partial x_1}, \dots, \frac{\partial f(\boldsymbol{x})}{\partial x_i})$

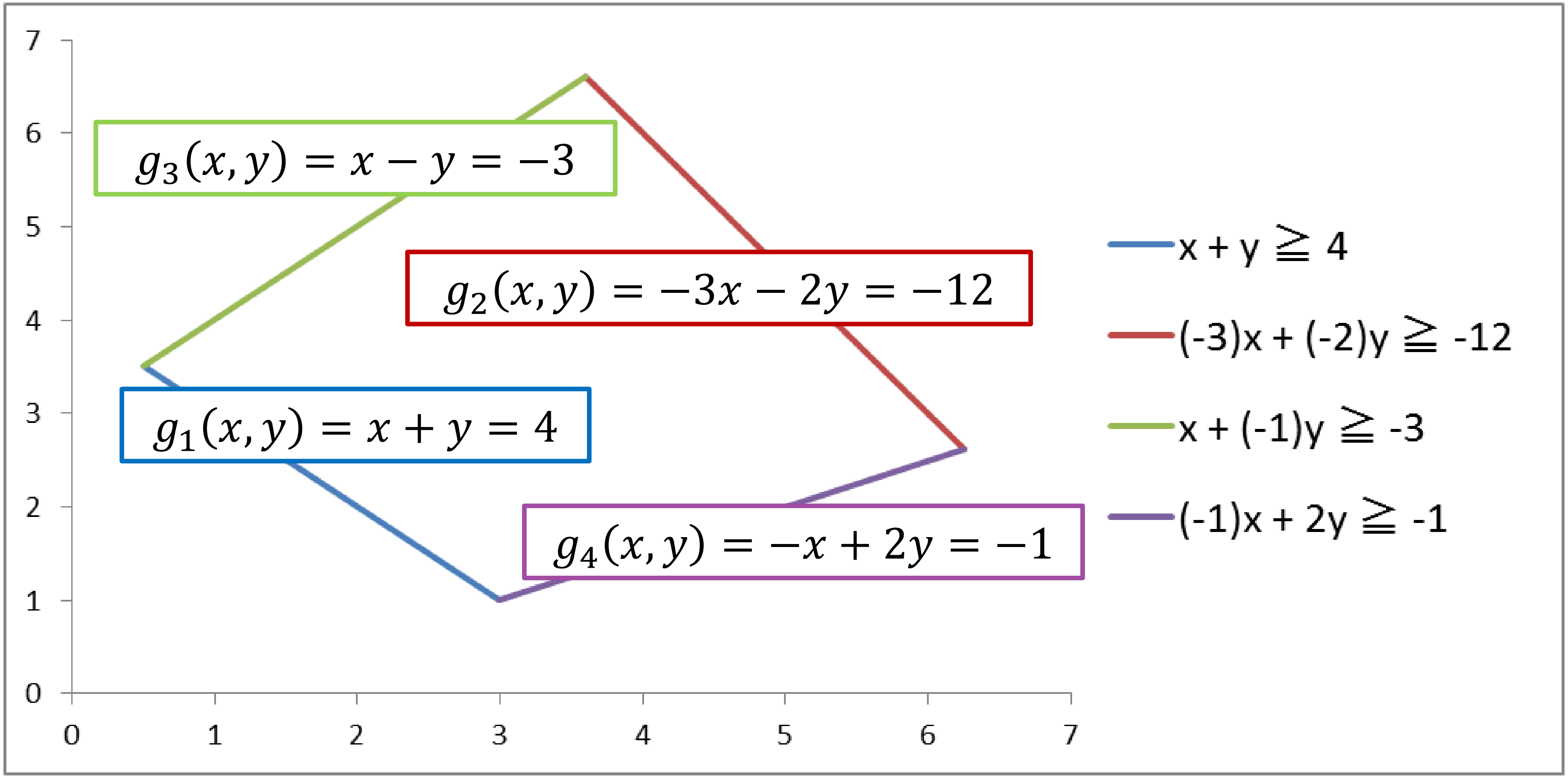
① $$\frac{\partial f(\boldsymbol{x}^*)}{\partial x_i} = \sum_j u_j * \frac{\partial g_j(\boldsymbol{x}^*)}{\partial x_i}$$

$$L(\boldsymbol{x}, \boldsymbol{u}) = f(\boldsymbol{x}) + \sum_{j} u_j * \left(\mathrm{b_j} - g_j(\boldsymbol{x})\right)$$ と置けば

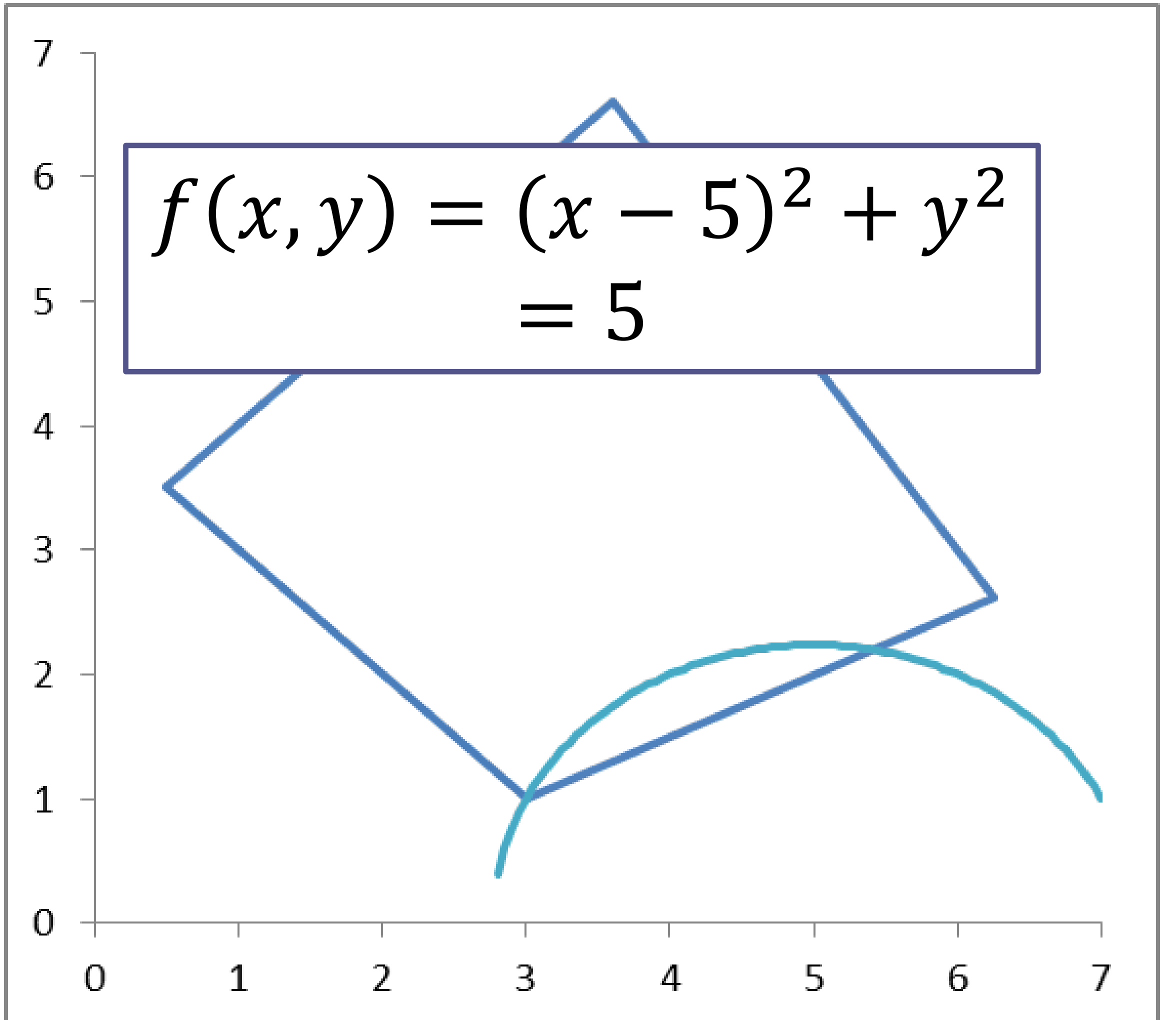
① $$\frac{\partial L(\boldsymbol{x}^*, \boldsymbol{u}^*)}{\partial x_i} = 0$$

③ $$u_j * \frac{\partial L(\boldsymbol{x}^*, \boldsymbol{u}^*)}{\partial u_j} = 0$$

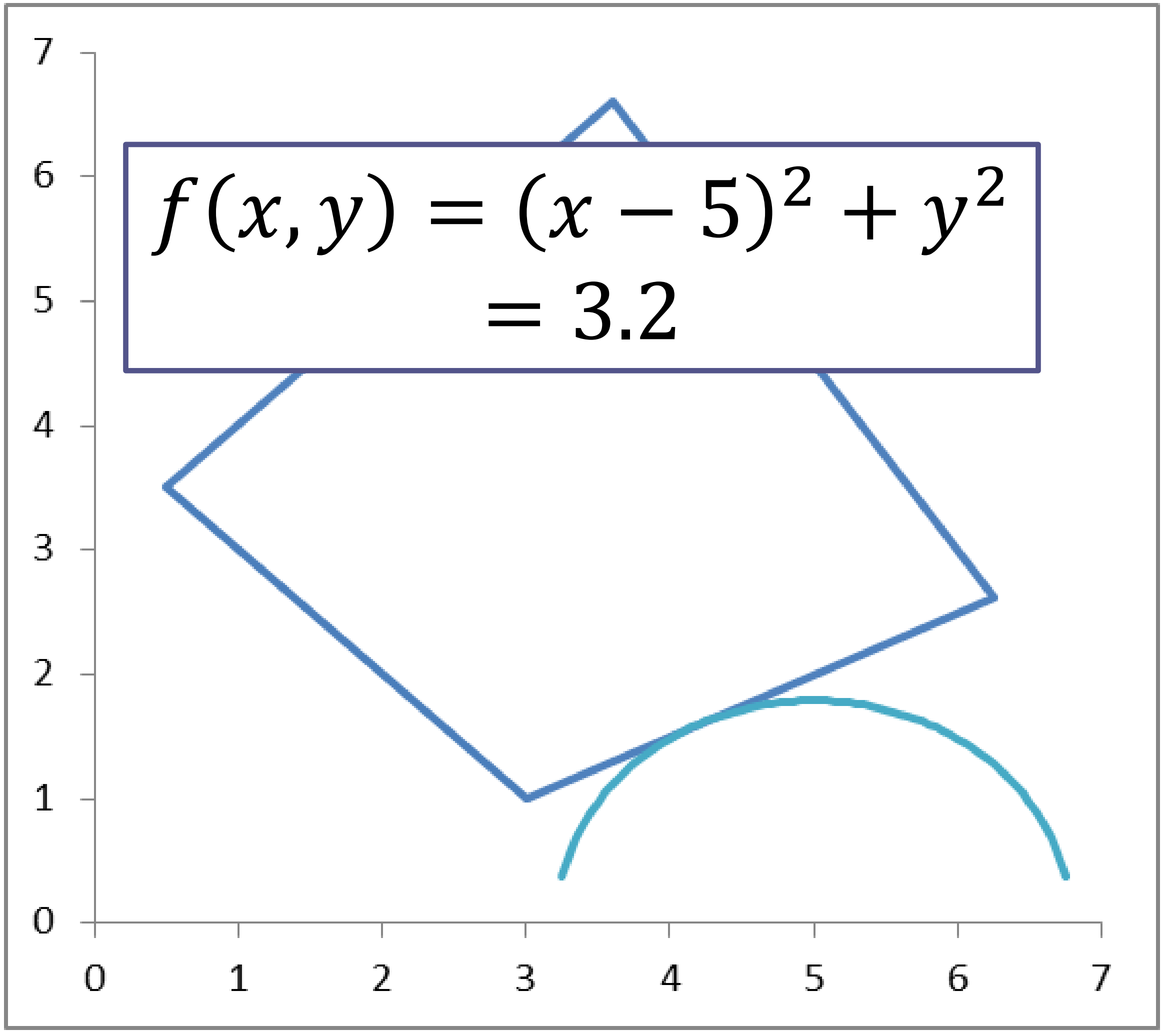
④ $$\frac{\partial L(\boldsymbol{x}^*, \boldsymbol{u}^*)}{\partial u_j} \leq 0$$



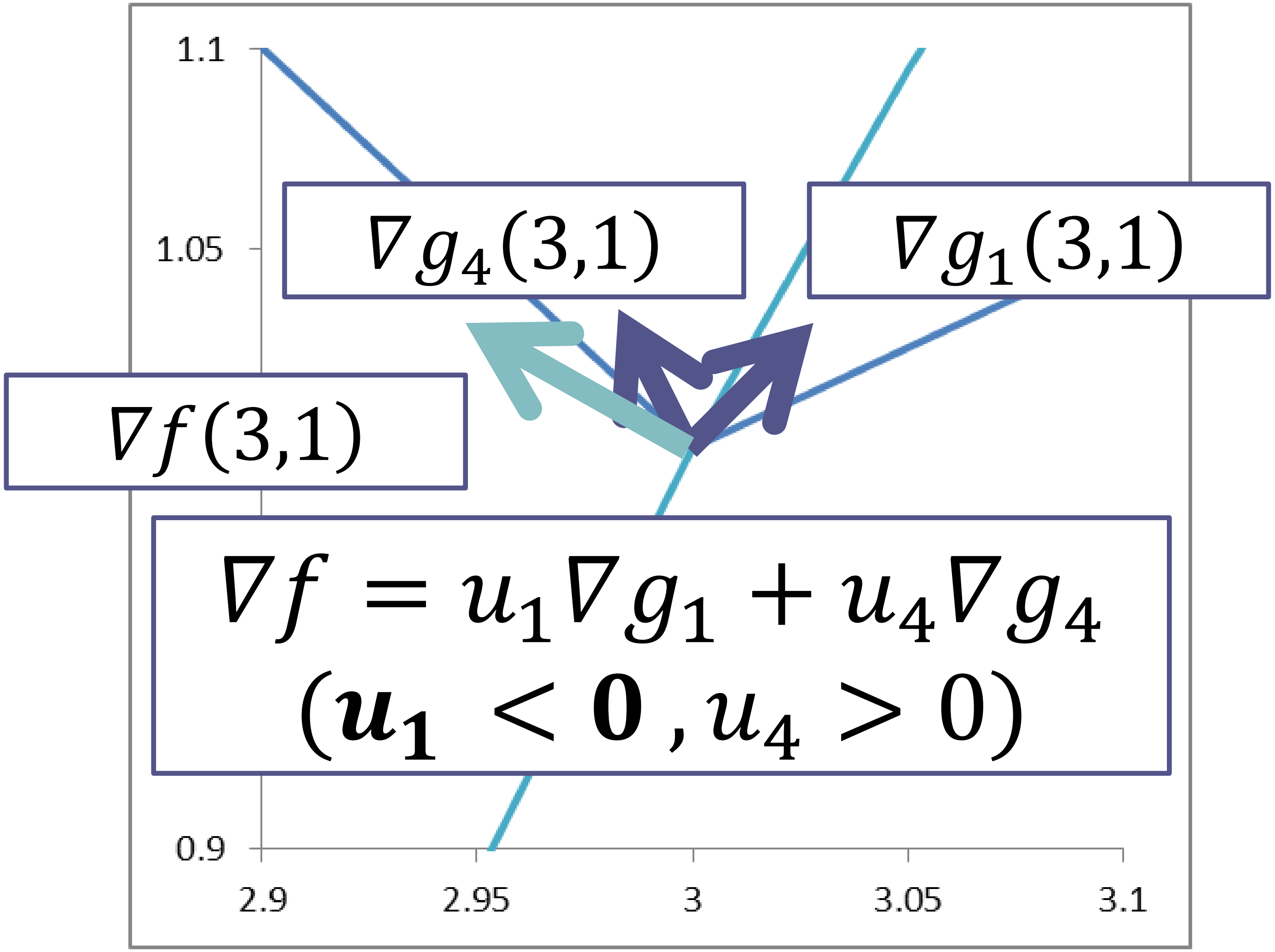
$$min.\ f(x,y) = (x-5)^2 + y^2$$



$$f(x, y) = (x - 5)^2 + y^2 = 5$$



$$f(x, y) = (x - 5)^2 + y^2 = 3.2$$



$$\nabla f = u_1 \nabla g_1 + u_4 \nabla g_4$$
$$(\boldsymbol{u_1} < \mathbf{0}\,, u_4 > 0)$$

3. KKTとLagrangeの比較

KKT

問題

$$max.\ f(\boldsymbol{x})$$

$$s.t.\ g_j(\boldsymbol{x}) \geq b_j$$

Lagrange

問題

$$max.\ f(\boldsymbol{x})$$

$$s.t.\ g_j(\boldsymbol{x}) = b_j$$

3. KKTとLagrangeの比較

KKT

必要条件

① $\frac{\partial f(\boldsymbol{x}^*)}{\partial x_i} = \sum_j u_j * \frac{\partial g_j(\boldsymbol{x}^*)}{\partial x_i}$

② $u_j \geq 0$

③ $u_j * \left(b_j - g_j(\boldsymbol{x}^*)\right) = 0$

④ $b_j - g_j(\boldsymbol{x}^*) \geq 0$

Lagrange

必要条件

① $\frac{\partial f(\boldsymbol{x}^*)}{\partial x_i} = \sum_j u_j * \frac{\partial g_j(\boldsymbol{x}^*)}{\partial x_i}$

② $b_j - g_j(\boldsymbol{x}^*) = 0$