Tokyo Tech

Optimization of inventory transportation strategy, considering the procurement of goods by the affected population

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21st Summer Course
Behavior Modeling in Transportation Networks

Tokyo Institute of Technology

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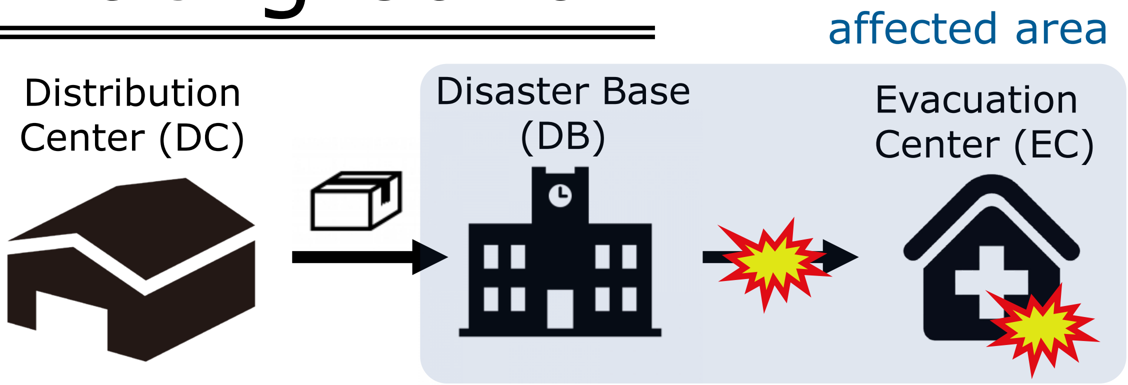
Background



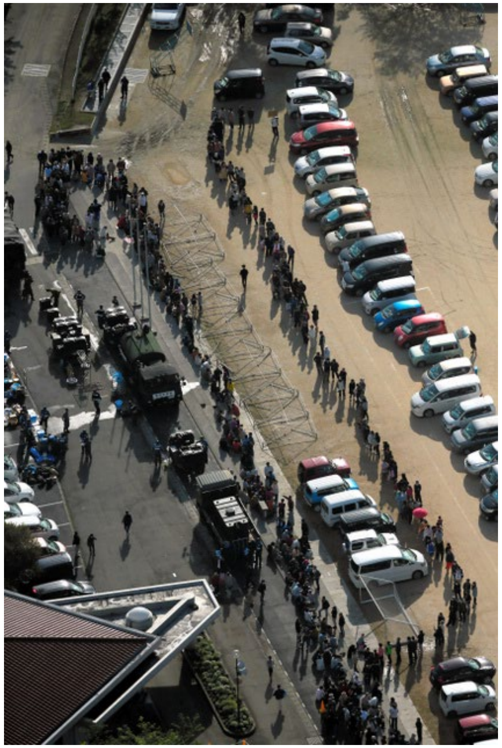
Relief Goods Transportation Network

- Humanitarian Organizations (HOs; e.g., governments) need to supply relief goods to ECs.

Bad Reports[3) (Examples)

- Surge Demand $\xrightarrow{\text{causes}}$ Queue Congestion
- Supply Constraints $\xrightarrow{\text{causes}}$ Inventory Congestion

Should rethink relief goods flow



(a) Queue by rationing [1)

(b) Inventory congestion [2)

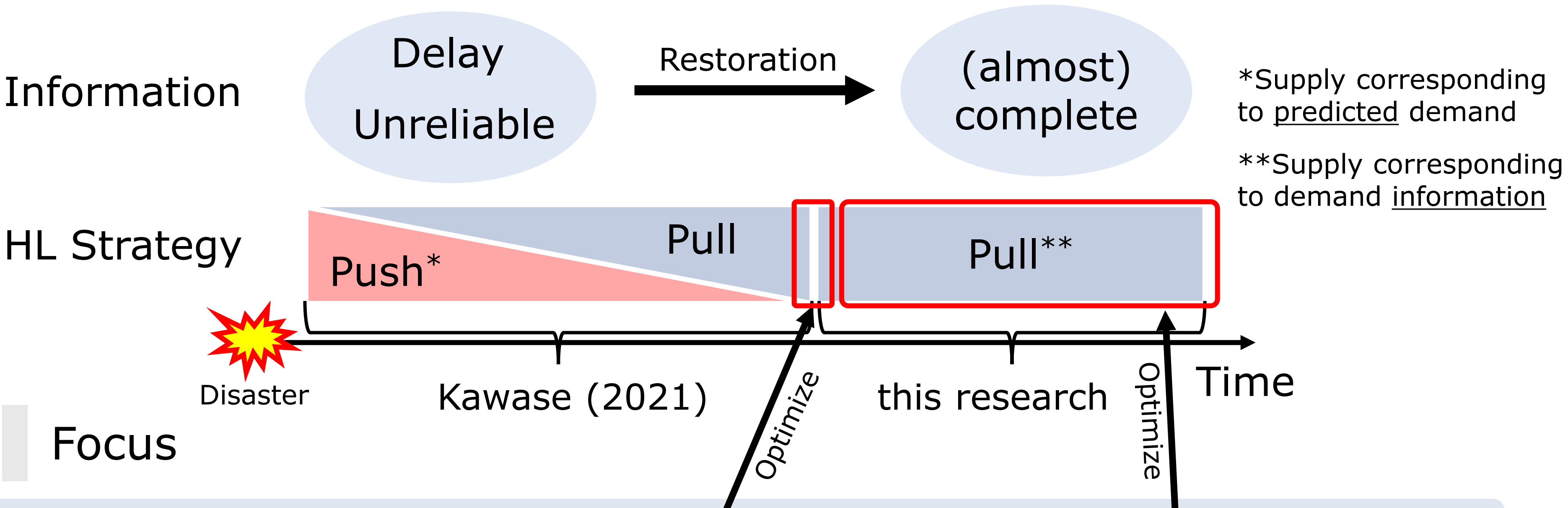
2016 Kumamoto Earthquakes

Humanitarian Logistics (HL)

Definition

" the process of planning, implementing and controlling ***the efficient, cost-effective flow and storage of goods and materials, as well as related information****, from the point of origin to the point of consumption for the purpose of alleviating the suffering of vulnerable people. "*

HL with restoration process of **Information** Infrastructure



*Supply corresponding to predicted demand

**Supply corresponding to demand information

Focus

What preparations (e.g., staffing) should be done for the pull strategy?

What is the efficient, cost-effective flow and storage in the pull strategy?

Challenges of Pull Strategy

(Almost) Perfect Information but **Supply Constrained**

- In the Kumamoto Earthquakes, direct supply was the solution alternative.
- However, a large-scale disaster would also disrupt direct supply.

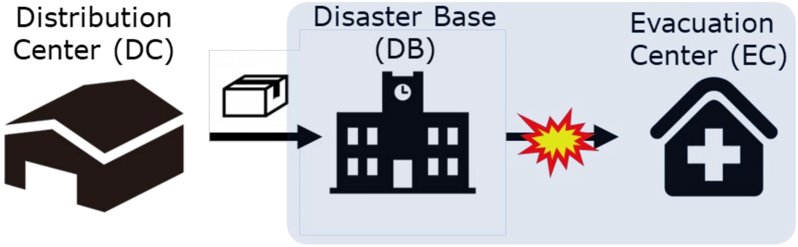
Observation (in the Kumamoto Earthquakes)

- **Some victims made trips** to acquire food and essential goods.

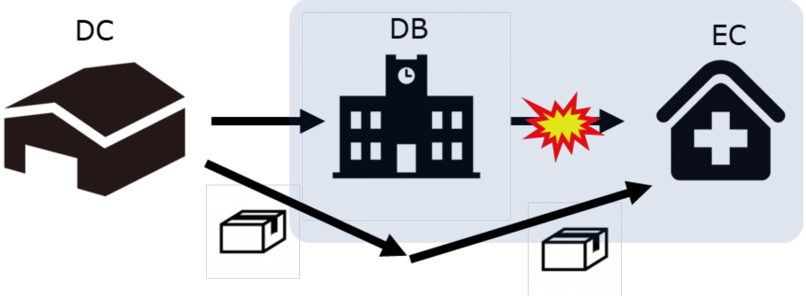
Suggest the potential for improvement through Demand-Supply Approach

Methodology

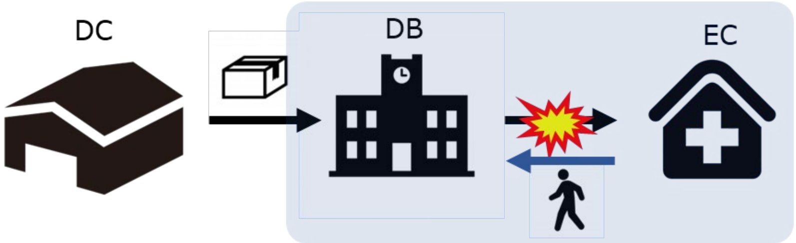
- Formulate an optimization problem for the demand-supply approach
- Numerical evaluate the performance of the approach



Supply Constrained



Direct Supply



Demand-Supply Approach

Previous Research

Previous models of network flow optimization for humans and goods

- ■ discard **Queue Congestion** and **Inventory Congestion**.
- ■ are **computationally inefficient**.

Typical Assumptions in Logistics Optimization

- ✓ Humans and Goods can move in free-flow travel time (FFTT).
- ✓ Humans/goods are serviced/consumed immediately on arrival.

Previous Models Considering Only Flow Congestion, not Queue Congestion

- ✓ Muggy and Stamm (2020) formulate MPEC* s.t. congestion game.
- ✓ Gutjahr and Dzubur (2016) formulate BLPP s.t. optimal destination choice.

Previous Models Considering Queue Congestion

- ✓ Espejo-Díaz and Guerrero (2021), Fikar et al.(2018) present an agent-based simulation optimization framework.
- ✓ Limitation is computational efficiency.

*Mathematical Programming with Equilibrium Constraints
**Bilevel Programming Problems

Previous Research

Research Gap

- Optimal network flow for humans (traffic) and goods could attack HL problems, supply constraints.
- There is no optimization model to meet both computationally efficient and queue congestion considerations.

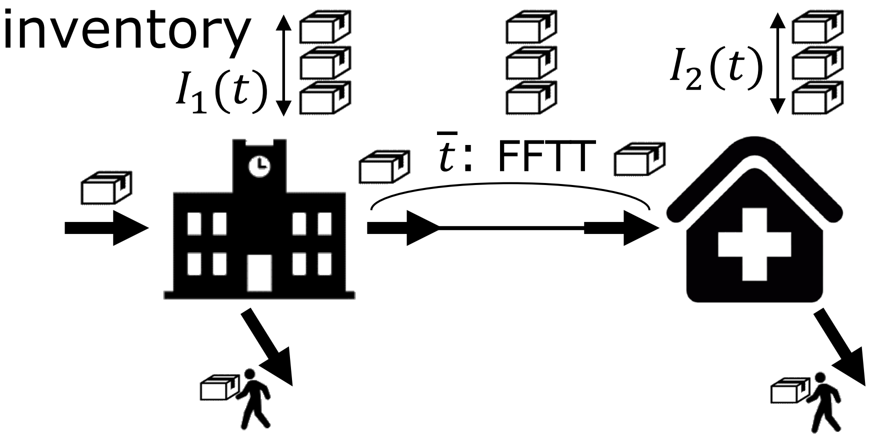
Research Objective

- Formulate an optimization problem that is (potentially) **computationally efficient** and describes **queue congestion**.
  - ✓ Propose BLPP
    - Upper: Optimize preparations (e.g., staffing, stockpiles)
    - Lower: Optimize dynamic network flow using Optimal Control Problem
- Numerical evaluate the performance of the **demand-supply approach**.
  - ✓ Show preliminary results on a simple network.

Modeling

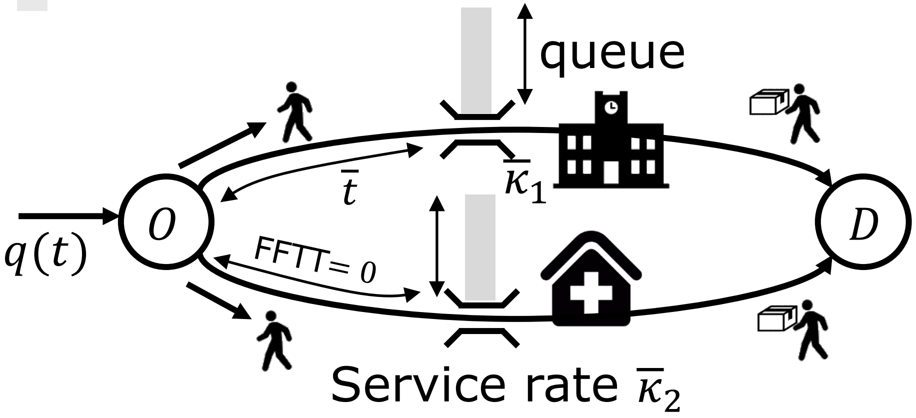
Model Framework

Relief Goods



Relief Goods Transportation Network

Human (Traffic, Victim)



A Parallel Link Network

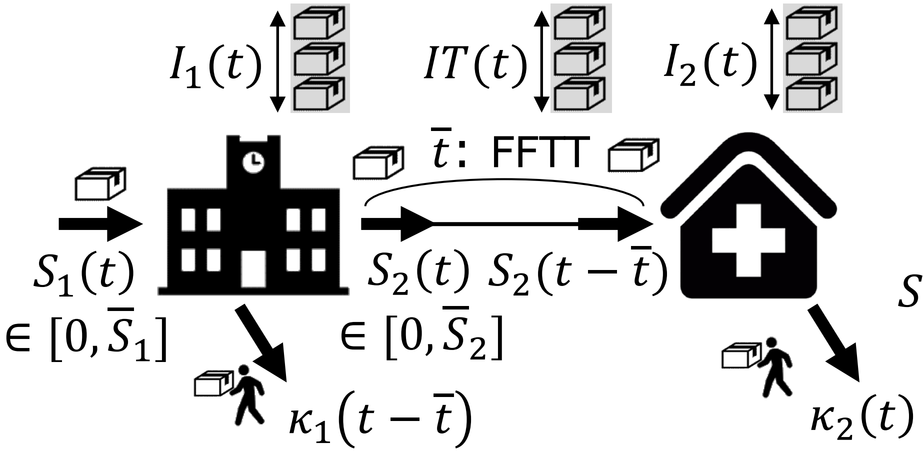
- To minimize queue congestion and inventory congestion,
  - ✓ (Upper) Determine preparation strategies $\{I_1(0), I_2(0), \bar{\kappa}_1, \bar{\kappa}_2\}$ and
  - ✓ (Lower) Determine demand and supply dynamics.

One victim needs only one item.

- (Supply) HOs supply DB, or supply EC via DB.
  - ✓ Between DB and EC, relief goods can move in FFTT.
- (Demand) Victims receive relief goods at DB or EC.
  - ✓ Between DB and EC, victims can move in FFTT.
  - ✓ Total relief demand for victims is uncertain. Denote the stochastic process as $Q(t)$ and the sample path as $q(t)$.

Variables

Relief Goods (c.f. Kawase et al., 2019)



Relief Goods Transportation Network

Stock (State Variables)

$I_1(t)$, $I_2(t)$: Net inventory* at nodes

$IT(t)$: In-transit inventory

Flow (Control Variables)

$S_1(t)$: Inflow per unit-time at DB

$S_2(t)$: Outflow per unit-time at DB

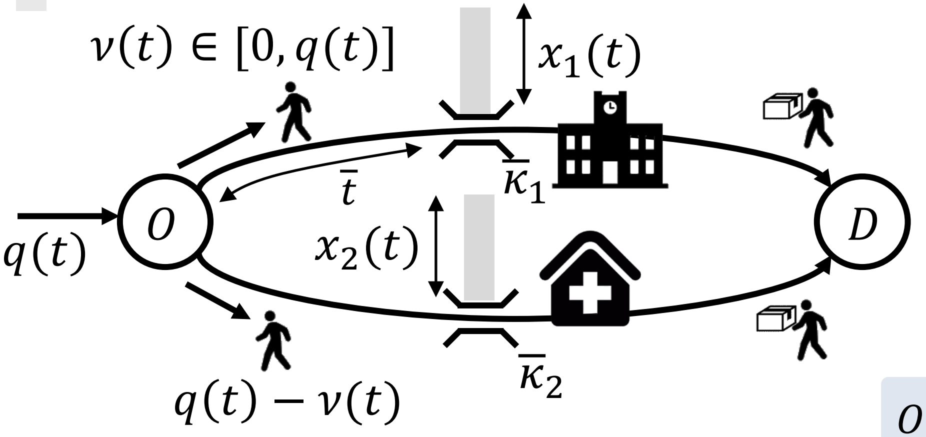
$S_2(t-\bar{t})$: Inflow per unit-time at EC

$\kappa_1(t), \kappa_2(t)$: Demand per unit-time

One victim needs only one item.

*an inventory if positive; a shortage otherwise

Human (c.f. Akamatsu and Nagae, 2007)



A Parallel Link Network

Stock (State Variables)

$x_1(t)$, $x_2(t)$: Queue

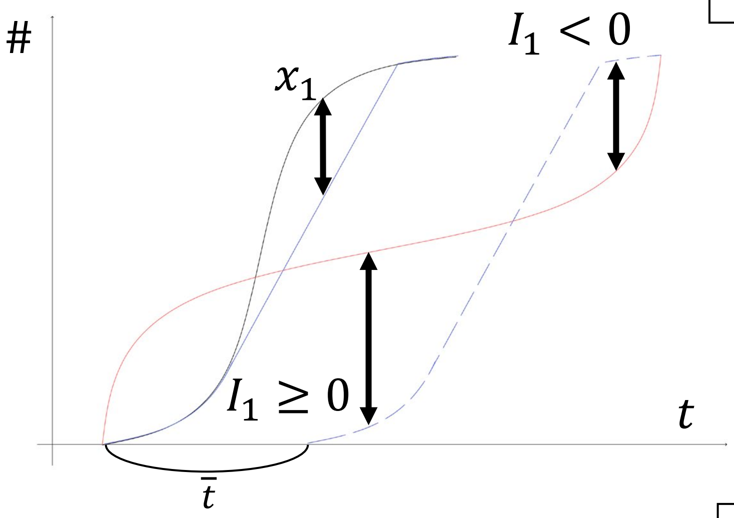
Flow (Control Variables)

$v(t)$: Inflow per unit-time to DB

$\kappa_1(t), \kappa_2(t)$: Outflow per unit-time

O: Need relief goods D: Get relief goods

Dynamics



Cumulative curve at DB

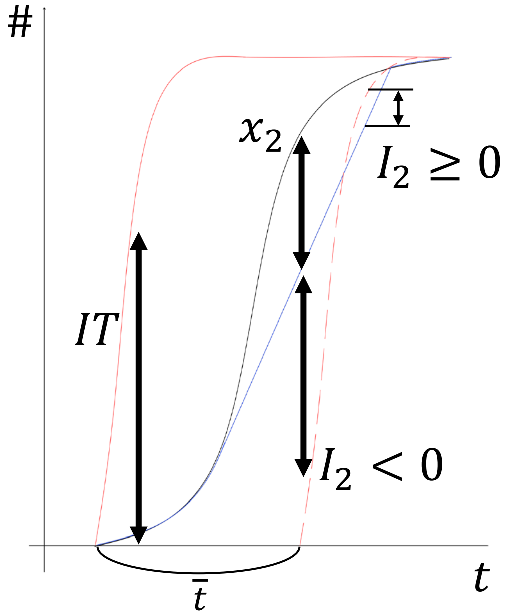
— $\int (S_1 - S_2)dt$ — $\int \kappa_1 dt$

— $\int \nu dt$ - - $\int \kappa_1(t - \bar{t})dt$

$$dI_1/dt = S_1(t) - S_2(t) - \kappa_1(t - \bar{t}),\quad I_1(0) = I_{01} \tag{1}$$

$$dx_1/dt = \nu(t) \qquad - \kappa_1(t), \qquad x_1(0) = 0 \tag{2}$$

$$\kappa_1(t) = \begin{cases} \bar{\kappa}_1 & \text{if } x_1(t) > 0 \\ \bar{\kappa}_1 & \text{if } x_1(t) = 0 \text{ and } \nu(t) \geq \bar{\kappa}_1 \\ \nu(t) & \text{if } x_1(t) = 0 \text{ and } \nu(t) < \bar{\kappa}_1 \end{cases} \tag{3}$$



Cumulative curve at EC

— $\int S_2 dt$ — $\int (q - \nu)dt$

- - $\int S_2(t - \bar{t})dt$ — $\int \kappa_2 dt$

$$dI_2/dt = S_2(t - \bar{t}) \qquad - \kappa_2(t), \qquad I_2(0) = I_{02} \tag{4}$$

$$dIT/dt = S_2(t) - S_2(t - \bar{t}), \qquad IT(0) = 0 \tag{5}$$

$$dx_2/dt = q(t) - \nu(t) \quad - \kappa_2(t), \qquad x_2(0) = 0 \tag{6}$$

$$\kappa_2(t) = \begin{cases} \bar{\kappa}_2 & \text{if } x_2(t) > 0 \\ \bar{\kappa}_2 & \text{if } x_2(t) = 0 \text{ and } q(t) - \nu(t) \geq \bar{\kappa}_2 \\ q(t) - \nu(t) & \text{if } x_2(t) = 0 \text{ and } q(t) - \nu(t) < \bar{\kappa}_2 \end{cases} \tag{7}$$

*(t) is omitted.

Cost

* This study assumes that the terminal cost $\phi(T)$ is 0.

Total Cost for a certain time horizon $[0, T]$ *

$$C = \int_0^T c(t)dt + \phi(T) \qquad (8)$$

Increase in Total Cost per unit-time, $c(t)$

$$c(t) = c_{I_1}(t) + c_{I_2}(t) + c_{IT}(t) + c_{x_1}(t) + c_{x_2}(t) \qquad (9)$$

Net Inventory Cost, $c_{I_1}(t) + c_{I_2}(t)$

$$c_{I_1}(t) = \max\{0, I_1(t)\} - \min\{0, I_1(t)\} \quad (10)$$

$$c_{I_2}(t) = \max\{0, I_2(t)\} - \min\{0, I_2(t)\} \quad (11)$$

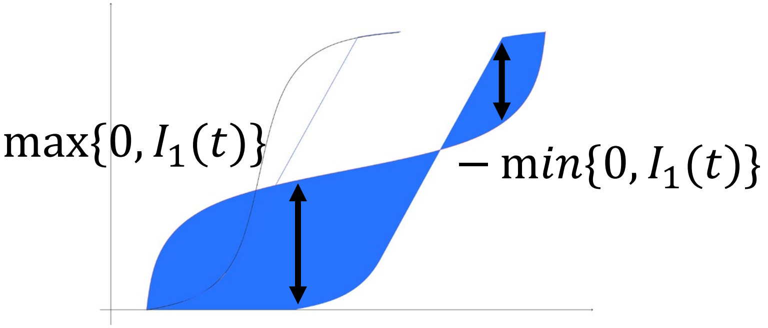
In-transit Inventory Cost, $c_{IT}(t)$

$$c_{IT}(t) = S_2(t) \cdot \bar{t} \qquad (12)$$

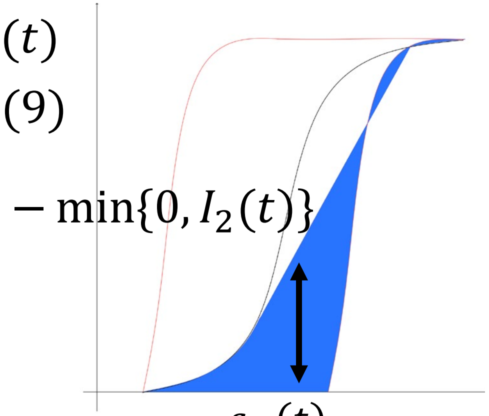
Total Travel Time, $c_{x_1}(t) + c_{x_2}(t)$

$$c_{x_1}(t) = \left(x_1(t)/\bar{\kappa}_1 + \bar{t}\right)v(t) \qquad (13)$$

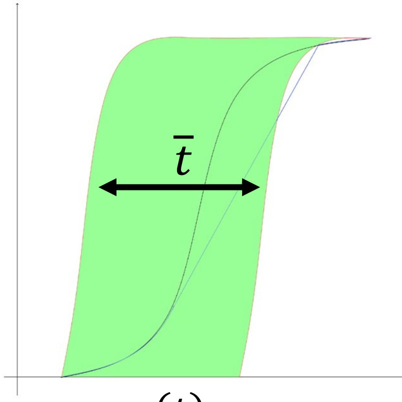
$$c_{x_2}(t) = x_2(t)/\bar{\kappa}_2(q(t) - v(t)) \quad (14)$$



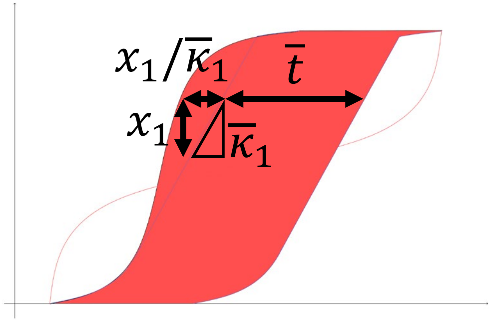
$c_{I_1}(t)$



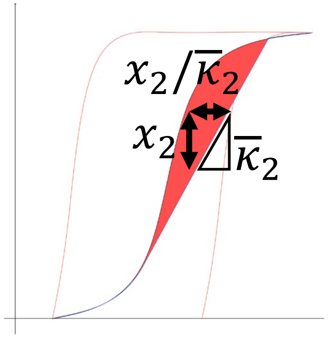
$c_{I_2}(t)$



$c_{IT}(t)$



$c_{x_1}(t)$



$c_{x_2}(t)$

Modified Cost

Problems involving delays are **not easy to solve**.

- Define modified cost $\hat{C}$ so that the problem does not include delays.
- When $I_1(t) \geq 0$ and $I_2(t) \geq 0\ \forall t \in [0, T]$, $C = \hat{C}$.

Modified Net Inventory Cost, $\hat{c}_{I_1}(t) + \hat{c}_{I_2}(t)$

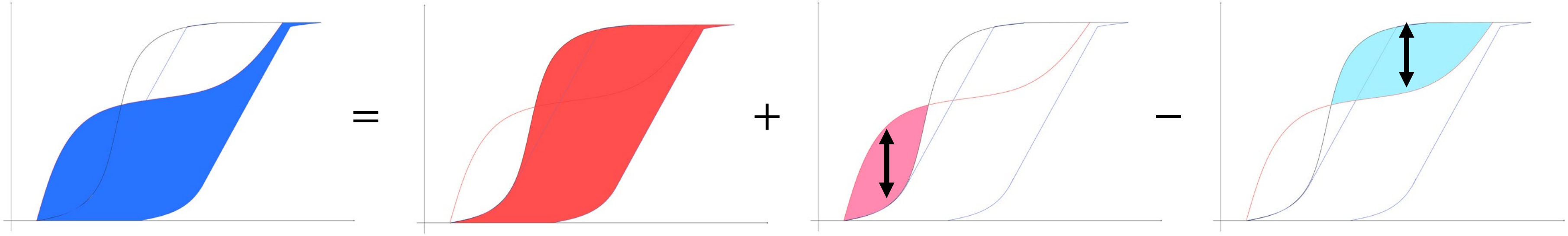
$$\hat{c}_{I_1}(t) = \max\{IN_1(t), 0\} - \min\{0, IN_1(t)\} + c_{x_1}(t) \quad (15)$$

$$\hat{c}_{I_2}(t) = IN_2(t) - c_{IT}(t) \quad (16)$$

$IN_1(t)$, $IN_2(t)$: Potential Net inventory, including unarrived supply and demand

$$dIN_1/dt = S_1(t) - S_2(t) - \nu(t), \quad IN_1(0) = I_{01} \quad (17)$$

$$dIN_2/dt = S_2(t) - \kappa_2(t), \quad IN_2(0) = I_{02} \quad (18)$$



$c_{I_1}(t)$ $\quad c_{x_1}(t)$ $\quad \max\{IN_1(t), 0\}$ $\quad \min\{0, IN_1(t)\}$

Bilevel Programming Problem

Lower-level: Linear Optimal Control Problem

$$\min_{\boldsymbol{X},\boldsymbol{U}} [\hat{C} | I_{01}, I_{02}, \overline{\kappa}_1, \overline{\kappa}_2, \{q(t)\}]$$

$\boldsymbol{X} := \{I_1(t),\ I_2(t), IN_1(t),\ IN_2(t), IT(t), x_1(t), x_2(t) | \forall t \in (0,T]\}$

$\boldsymbol{U} := \{S_1(t), S_2(t), \nu(t), \kappa_1(t), \kappa_2(t) | \forall t \in [0,T)\}$

s.t. $\quad$ Eq. (1) – (7), (12) – (18)

$$\hat{C} = \int_0^T \hat{c}(t)dt \qquad \hat{c}(t) = \hat{c}_{I_1}(t) + \hat{c}_{I_2}(t) + c_{IT}(t) + c_{x_1}(t) + c_{x_2}(t)$$

- Given preparation strategies $\{I_{01}, I_{02}, \overline{\kappa}_1, \overline{\kappa}_2\}$ and a demand sample path $\{q(t)\}$, the optimal control is obtained numerically.
- Apply existing computationally efficient solution method (c.f. Chavanasporn and Ewald, 2012) for linear optimal control problem.

Upper-level

$$\min_{I_{01}, I_{02}, \overline{\kappa}_1, \overline{\kappa}_2} \mathrm{E}[\hat{C} | \boldsymbol{X}, \boldsymbol{U}] \quad \text{s.t.} \quad \text{Lower-level Problem and}$$

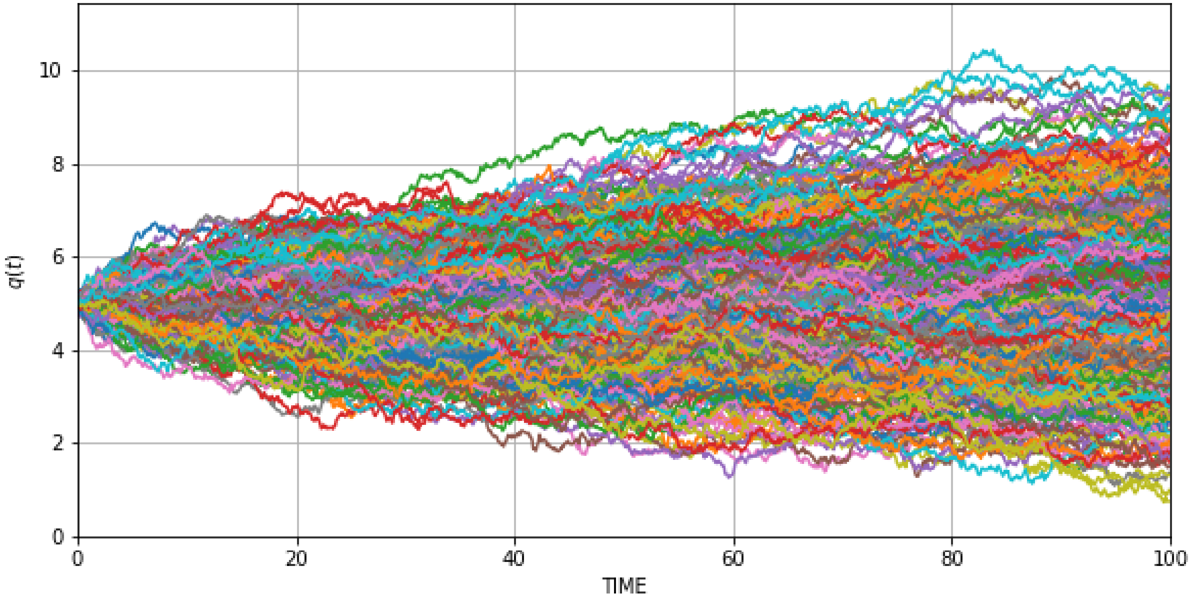
$$I_1(t) \geq 0, I_2(t) \geq 0,\ \ \overline{\kappa}_1 + \overline{\kappa}_2 = \overline{\kappa},\ \ \{Q(t)\}$$

- Generate a finite number of sample paths $q(t)$ of stochastic process $Q(t)$
- Evaluate the expected total cost approximately
- Find the optimal $I_{01}, I_{02}, \overline{\kappa}_1, \overline{\kappa}_2$ in a brute-force fashion

Numerical Experiments

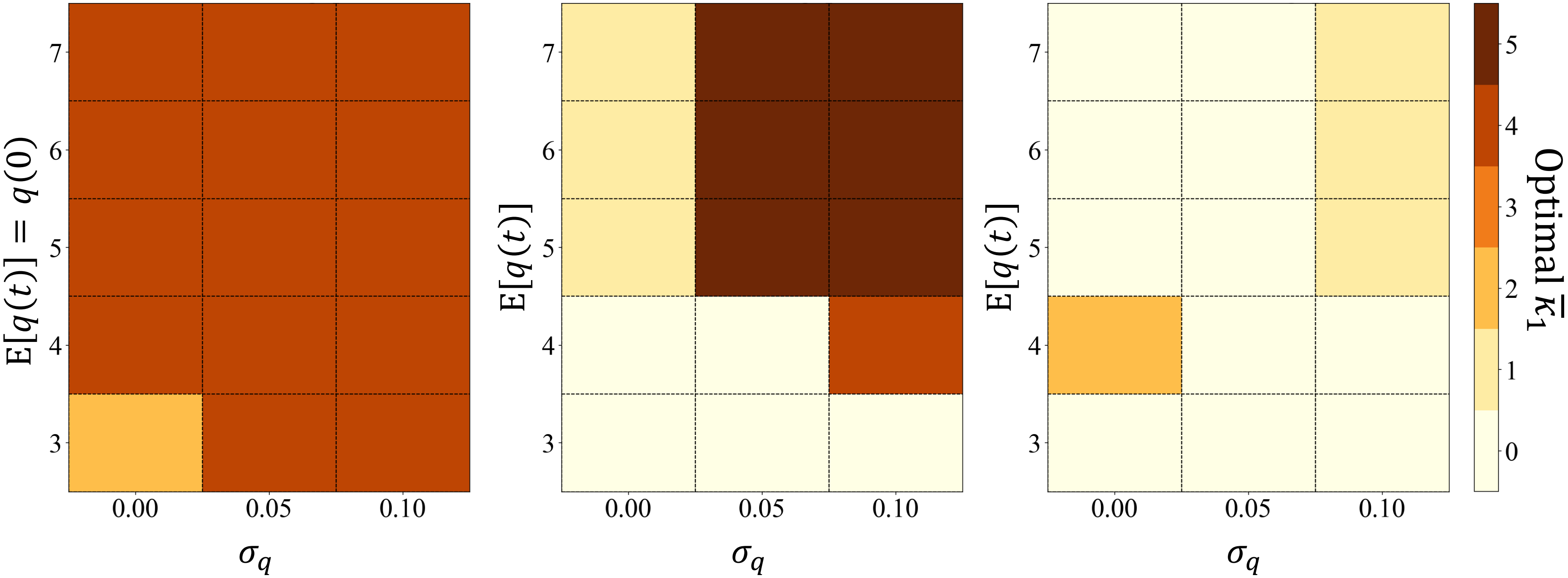
Numerical Experiments: Setting

- The number of sample paths $q(t)$ is 500.
- $dq(t) = \sigma_q dz(t)$, $z(t) \sim N(0, t)$, $q(0) = \{3,4,5,6,7\}$, $\sigma_q = \{0, 0.05, 0.1\}$
- $T = 100$
- FFTT, $\bar{t} = \{5,10,20\}$
- $\bar{S}_1 = 10, \bar{S}_2 = \{3,5,7\}$
- $\bar{\kappa} = 5, \bar{\kappa}_1 = \{0,1,2,3,4,5\}$



500 Sample Paths ($q(0) = 5$, $\sigma_q = 0.05$, then $q(t) \sim N(q(0), \sigma_q^2 t)$)

Numerical Experiments: Result

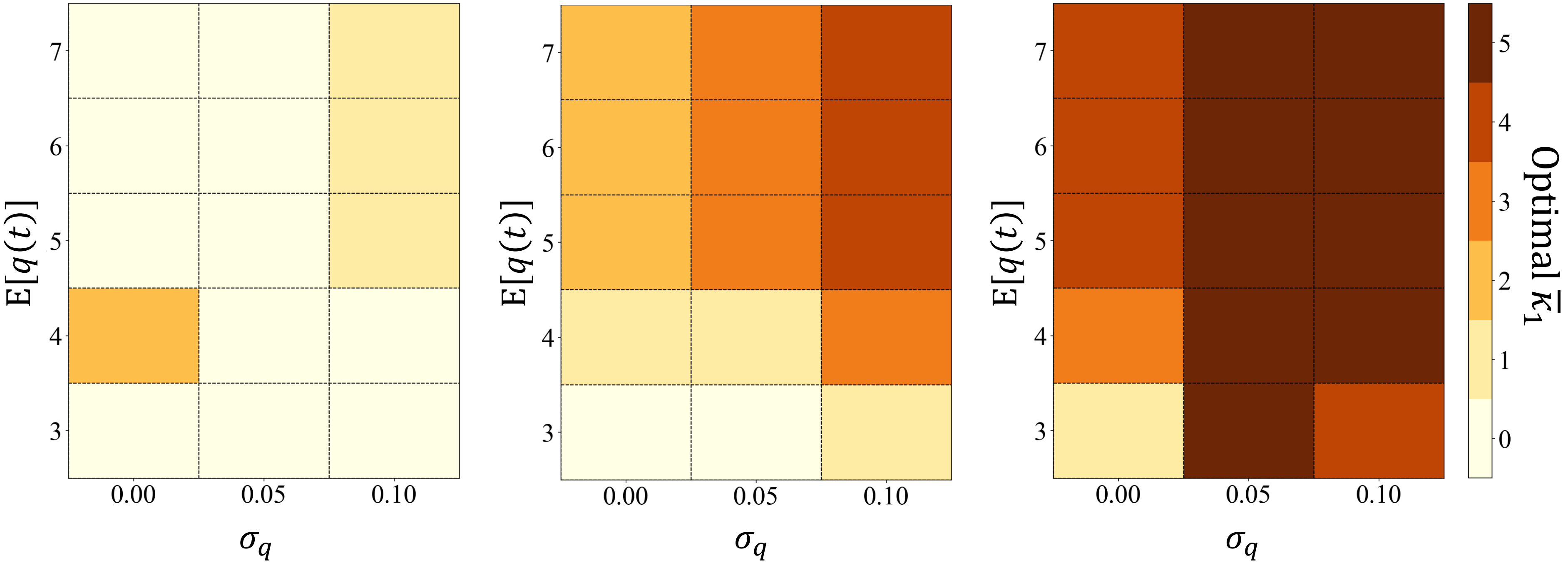


Optimal $\bar{\kappa}_1$ when FFTT $\bar{t} = 5$ (left: $\bar{S}_2 = 3$, center: $\bar{S}_2 = 5$, right: $\bar{S}_2 = 7$)

- The larger mean $\mathrm{E}[q(t)]$ and variance $\sigma_q^2 t$ of demand increase optimal $\bar{\kappa}_1$.
- When supply constraint is relaxed ($\bar{S}_2 = 7$), optimal $\bar{\kappa}_1$ is likely to be 0.

Suggest that in a typical disaster situation
($\mathrm{E}[q(t)]$, $\sigma_q^2 t$ are large, supply constraint is tight),
optimal $\bar{\kappa}_1 > 0$ (i.e., demand-supply approach is effective).

Numerical Experiments: Result



Optimal $\bar{\kappa}_1$ when $\bar{S}_2 = 7$(left: FFTT $\bar{t} = 5$, center:$\bar{t} = 10$, right:$\bar{t} = 20$)

- FFTT $\bar{t}$ between DB and EC increases, then
  - ✓ optimal $\bar{\kappa}_1$ tends to increase, and
  - ✓ the ratio of users' cost (total travel time for victims) increase.
  - ✓ Non-negative constraints on $I_1(t)$ and $I_2(t)$ could be causing supply-side costs to be greater than optimum.

Conclusions

- ■ Formulation
  - ✓ Formulate BLPP to evaluate the performance of demand-supply approach
    - Upper: Optimize preparations (e.g., staffing, stockpiles)
    - Lower: Optimize dynamic network flow for humans and goods
  - ✓ Reformulate lower-level one as a (non-delayed) optimal control problem by modifying net inventory cost

- ■ Numerical Experiments
  - ✓ **Demand-supply approach would be effective in a typical disaster situation.**
  - ✓ However, the proposed model is likely to overestimate supply-side costs. The optimality of the reformulated problem, optimal control problem, should be evaluated.

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Case Studies

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Optimal Control Problem

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