Structural estimation for a route choice model with uncertain measurement

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Outline

1. Introduction
2. Link-based route measurement model
3. Structural estimation method
4. Numerical examples
5. Conclusions
Outline

1. Introduction
2. Link-based route measurement model
3. Structural estimation method
4. Numerical examples
5. Conclusions
Route choice analysis

Parameter estimation results largely depend on accuracy of route measurement

GPS data

Route measurement model

Route choice data

Route choice model

Estimated parameters
Parameter estimation results largely depend on accuracy of route measurement.

GPS data → Route measurement model → Route choice data

Route choice model → Estimated parameters
Pedestrian route choice analysis

Measurement uncertainty; Dense and high-resolution network

- Dense network
- Spatial dependence of Measurement errors
  - Along river
  - Wide street
  - Narrow street
  - With arcade

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Route measurement models (1)

Sequential approach infers the true location at each data in chorological order

\[ \times: \text{GPS data} \]
Route measurement models (1)

*Sequential approach* can output *meaningless paths*
Route measurement models (2)

*Path-based probabilistic approach* evaluates *path likelihood regarding all GPS data included in a trip*

Pyo et al. (2001); Bierlaire et al. (2013)
Route measurement models (2)

*Path-based probabilistic approach* suffers with *trade-off* between computational efficiency and measurement accuracy

Pyo et al. (2001); Bierlaire et al. (2013)
Route measurement models (2)

Assuming error variance constant on network distort measurement probabilities

PDF of GPS measurement error:

\[ p(\hat{x}_j \mid x_j; \sigma) = \frac{\|\hat{x}_j - x_j\|}{\sigma^2} \exp \left( -\frac{\|\hat{x}_j - x_j\|^2}{2\sigma^2} \right) \]

\[ \sigma : \text{Large} \quad \sigma : \text{Small} \]

Distance from the true location

Measurement probability

\[ \|\hat{x}_j - x_j\| = d \]

\[ \sigma = 10 \]

\[ \sigma = 50 \]

\[ \sigma = 100 \]
Route measurement models (3)

Bayesian approach incorporates behavioral models into measurement models

Chen et al. (2013); Danalet et al. (2014)

Prior (Route choice model)

- Prior: 25%
- 15%
- 60%

Measurement

- Measurement: 10%
- 40%
- 50%

Posterior

- Posterior: 16%
- 6%
- 78%
Bayesian approach has a problem regarding parameter inconsistency.

Route measurement models (3)

GPS data → Route measurement model → Route choice data → Route choice model → Estimated parameters

Given parameters:
- Arbitrary
- Previous study
- Other data
Route measurement models

• **Challenges:**
  - *Disconnected path*
    - Not suitable for route choice models
  
  - *Setting of the measurement parameter*
    - Possible to miss the true path
    - Ignorance of spatial difference distorts path likelihood
  
  - *Parameter inconsistency of route choice model*
    - Estimated parameter includes biases regarding initial parameter
1. **Link-based route measurement model**
   - Matching each decomposed trip data to a link
   - Estimating a measurement parameter for each link
   - Incorporating a *link-based route choice model* as prior

2. **Structural estimation method**
   - Parameters at convergence **satisfy the parameter consistency** of the route choice model
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Problem & Notation

Matching row GPS data $\hat{m}$ to the transportation network $G$

- **GPS data** $\hat{m} = (\hat{x}, \hat{\tau})$
  - Pair of coordinates $\hat{x} = (\hat{x}_{lat}, \hat{x}_{lon})$ with error variance $\sigma$
  - Timestamp $\hat{\tau}$
  - A given trip $\hat{m} = (\hat{m}_1, ..., \hat{m}_n, ..., \hat{m}_N)$

- **Network** $G = (V, A)$
  - Node $v \in V$: the horizontal position $x_v = \{x_{lat}, x_{lon}\}$
  - Link $a = (v_u, v_d) \in A$: the vector of spatial attributes $y_a$
  - Network connection $\delta(a'|a) : 1/0$
Link-based route measurement

Matching all data observed within a period to the same link

GPS data of a trip
\[ \hat{\mathbf{m}} = (\hat{m}_1, \ldots, \hat{m}_n, \ldots, \hat{m}_N) \]
\[ \hat{m}_n = (\hat{x}_n, \hat{\tau}_n) \]

Data decomposition
\[ \hat{\mathbf{m}} = (\hat{m}_1, ..., \hat{m}_t, ..., \hat{m}_j) \]
\[ \hat{m}_t = (\hat{m}_1, ..., \hat{m}_j, ..., \hat{m}_j) \]

Estimation: \( \sigma_{a_t} \)

Link probabilities

Path inference:
\[ \psi = [a_1, ..., a_t, ..., a_T] \]

\[ p(\hat{m}_t | a_t, a_{t-1}) \propto p(\hat{m}_t | a_t; \sigma_{a_t}) p(a_t | a_{t-1}; \theta) \]

\[ p(\hat{m}_t | a_t) \] : Measurement probability of \( \hat{m}_t \) given \( a_t \); **measurement equation**

\[ p(a_t | a_{t-1}) \] : Prior probability of \( a_t \) given \( a_{t-1} \) ; **system equation**
**Link probability** \( p(a_t | \hat{m}^t, a_{t-1}) \)

The probability of \( a_t \) given measurements \( \hat{m}^t \) and state \( a_{t-1} \)

- **Candidate set:** \( A(a_{t-1}) = \{a_t | \delta(a_t | a_{t-1}) = 1\} \)
  - Calculate link probabilities for all \( a_t \in A(a_{t-1}) \)

\[ \hat{m}_1^t, \hat{m}_2^t, \hat{m}_3^t \]

\( \times \) : GPS data \( \hat{m}^t = \{\hat{m}_1^t, \hat{m}_2^t, \hat{m}_3^t\} \)
Measurement equation \( p(\hat{m}^t \mid a_t; \sigma_{a_t}) \)

The probability of measurements \( \hat{m}^t \) given \( a_t \)

- **Assumption:**
  - Timestamp \( \hat{t} \) has no measurement error; \( p(\hat{m}^t \mid a_t) = p(\hat{x}^t \mid a_t) \)
  - Measurement probability of data is independent from each other
  - Traveler moves at the constant speed on the same link

\[
p(\hat{x}_1^t, \ldots, \hat{x}_J^t \mid a_t; \sigma_{a_t}) = \prod_{j=1}^J p(\hat{x}_j^t \mid a_t; \sigma_{a_t})
\]

\[
= \prod_{j=1}^J \int_{x_j \in a_t} p(\hat{x}_j^t \mid x_j^t, a_t; \sigma_{a_t}) p(x_j \mid a_t) dx_j
\]

PDF of GPS measurement error: *Rayleigh distribution* (van Diggelen, 2007)

\[
p(\hat{x}_j^t \mid x_j^t, a_t; \sigma_{a_t}) = \frac{\|\hat{x}_j^t - x_j\|}{\sigma_{a_t}^2} \exp\left( - \frac{\|\hat{x}_j^t - x_j\|^2}{2\sigma_{a_t}^2} \right)
\]
Estimation of measurement parameter $\sigma_{a_t}$

Link-based map matching can regard error variance as a link peculiar variable

Measurement likelihood maximization

$$\sigma_{a_t} = \text{argmax}_{\sigma} p(\hat{\mathbf{m}}^t \mid a_t; \sigma)$$

Where,

$$p(\hat{\mathbf{m}}^t \mid a_t; \sigma_{a_t}) = \prod_{j=1}^{J} \int_{x_j \in a_t} \left\{ \frac{\| \hat{x}_j^t - x_j \|}{\sigma_{a_t}^2} \exp \left( - \frac{\| \hat{x}_j^t - x_j \|^2}{2\sigma_{a_t}^2} \right) \right\} \cdot p(x_j \mid a_t) \, dx_j$$
System equation $p(a_t | a_{t-1}; \theta)$

The prior probability of $a_t$ given a state $a_{t-1}$

**Link-based route choice model**

Utility function:

$$u(a_t | a_{t-1}) = v(a_t | a_{t-1}) + \varepsilon(a_t) = \theta y_{a_t|a_{t-1}} + \varepsilon(a_t)$$

- $v(\cdot)$: Deterministic component of utility
- $\varepsilon(\cdot)$: Probabilistic component of utility
  (i.i.d. gumbel distribution)
- $y_{a_t|a_{t-1}}$: Vector of explanatory variables
- $\theta$: Vector of parameters
System equation \( p(a_t \mid a_{t-1}; \theta) \)

The prior probability of \( a_t \) given a state \( a_{t-1} \)

Link-based route choice model

Choice probability option:

**Markov model**

\[
p(a_t \mid a_{t-1}) = \frac{e^{v(a_t \mid a_{t-1})}}{\sum_{a_t \in A(a_{t-1})} e^{v(a_t \mid a_{t-1})}}
\]

**Recursive logit model** Fosgerau et al. (2013)

\[
p(a_t \mid a_{t-1}) = \frac{e^{v(a_t \mid a_{t-1}) + V^d(a_t)}}{\sum_{a_t \in A(a_{t-1})} e^{v(a_t \mid a_{t-1}) + V^d(a_t)}}
\]

And others: e.g.,

Mai et al. (2015); Mai (2016); Oyama et al. (2016)
Link inference

- **Link (posterior) probability:**
  - The probability of $a_t$ given measurements $\hat{m}^t$ and a state $a_{t-1}$

  \[ p(a_t | \hat{m}^t, a_{t-1}) \propto p(\hat{m}^t | a_t; \sigma_{a_t}) p(a_t | a_{t-1}; \theta) \]

- **Link inference:**
  - Link likelihood maximization subject to switching condition

  \[ a_t = \arg\max_{a_t \in A(a_{t-1})} p(a_t | \hat{m}^t, a_{t-1}) \]

  \[ \text{s.t., } \max_{a \in A(a_t)} p(\hat{m}^{t+1} | a; \sigma_a) > \gamma \]
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A fixed point problem

Need to solve a fixed point problem of route measurement and estimation

Route measurement

Route choice probability

\[ p(a_t | a_{t-1}; \tilde{\theta}) \]

Path \( \psi \)

Measurement probability

\[ p(\hat{m}_t | a_t; \sigma_{a_t}) \]

Route choice model

\[ \theta = \arg\max_{\theta} LL(\theta; \psi) \]

s.t., \( |\theta - \tilde{\theta}| < \zeta \)

Where,

\[ LL(\theta; \psi) = \ln \left( \prod_{i} \prod_{t=2}^{T} p(a_t | a_{t-1}; \theta) \delta_{\psi} \right) \]

\[ \delta_{a_t}^i = \begin{cases} 1, & \text{if } a_{i,t} \in \psi_i \\ 0, & \text{otherwise} \end{cases} \]
Structural estimation

A method for parameter estimation of models with fixed point problem

- NFXP (Nested Fixed Point)  Rust (1987)
- NPL (Nested Pseudo Likelihood)  Aguirregabiria and Mira (2002)
- MPEC (Mathematical Programming with Equilibrium Constraint)  Su and Judd (2012)
- ...

Structural estimation for route choice model with uncertain data

- Solving a fixed problem regarding parameter of route choice model
- Inner problem: Route measurement model
- Outer problem: Parameter estimation of route choice model
Structural estimation

A estimation method for solving a fixed point problem of route choice parameter

\[ \tilde{\theta} \]

\[ h = 1, \quad \theta^{(h)} = \tilde{\theta} \]

GPS data \( \hat{m} \)

**Measurement model**

\[
p_i(a_t | \hat{m}_t, a_{t-1}) = \frac{p(\hat{m}_t | a_t; \sigma_a) p(a_t | a_{t-1}; \theta^{(h)})}{\sum_{a_t} p(\hat{m}_t | a_t; \sigma_a) p(a_t | a_{t-1}; \theta^{(h)})}
\]

\[ \hat{m}_t = (\hat{m}_t^1, ..., \hat{m}_t^T) \]

\[ a_{i,t} = \arg\max_{a_t} p_i(a_t | \hat{m}_t^i, a_{t-1}; \theta^{(h)}) \]

\[ \psi_i^{(h)} = [a_{i,1}, ..., a_{i,t}, ..., a_{i,T}] \]

**Behavior model**

\[
\max_{\theta} LL(\theta) = \log \left( \prod_{i} \prod_{t=2}^{T} p_i(a_t | a_{t-1}; \theta)^{\delta_{a_{i,t}}} \right)
\]

\[ \delta_{a_{i,t}}^{(h)} = \begin{cases} 1, & \text{if } a_{i,t} \in \psi_i^{(h)} \\ 0, & \text{otherwise} \end{cases} \]

\[ \theta^{(h+1)} = \arg\max_{\theta} LL(\theta; \psi^{(h)}) \]

\[ h := h+1 \]

No

\[ |\theta^{(h+1)} - \theta^{(h)}| < \zeta \]

Yes

\[ \theta = \theta^{(h)}, \psi = \psi^{(h)} \]
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Twins experiments | Simulation

**Settings**

\[ v(a \mid k) = \theta_1 TT_a + \theta_2 CC_a + \theta_3 DC_a + \theta_4 UT_{a,k} \]

True parameter: \( \hat{\theta} = [-0.1, -2, -1.5, -4] \)

Period interval: \( \tilde{\tau} = 30s \)

Data generation: \( \hat{\tau}_j - \hat{\tau}_{j-1} = 10s \)

*(continuous cost: \( CC_a \) / discrete cost: \( DC_a \) / variance: \( \sigma_a \) )

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**Twins experiments | Measurement results**

*Which model improves the route measurement accuracy?*

<table>
<thead>
<tr>
<th>Model</th>
<th>σ</th>
<th>θ</th>
<th>accuracy (%)</th>
<th>Ave.</th>
<th>Switching</th>
<th>Switching</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MEQ</td>
<td>given</td>
<td>-</td>
<td>54.571</td>
<td>68.857</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2 MEQ</td>
<td>estimated</td>
<td>-</td>
<td>76.857</td>
<td>82.857</td>
<td>5.848</td>
<td>4.397</td>
</tr>
<tr>
<td>3 MEQ+SEQ</td>
<td>estimated [0, 0, 0, 0]</td>
<td>76.857</td>
<td>82.857</td>
<td>5.848</td>
<td>4.397</td>
<td></td>
</tr>
<tr>
<td>4 MEQ+SEQ</td>
<td>estimated [-1.5, -0.1, -2, -10]</td>
<td>4.857</td>
<td>38.286</td>
<td>41.992</td>
<td>21.206</td>
<td></td>
</tr>
<tr>
<td>5 MEQ+SEQ</td>
<td>estimated [-0.1, -2, -1.5, -4]</td>
<td>76.857</td>
<td>91.714</td>
<td>7.579</td>
<td>4.056</td>
<td></td>
</tr>
</tbody>
</table>

*MEQ: Measurement Equation
*SEQ: System Equation
Twins experiments | Estimation results

Does **structural estimation method** improve the parameter estimation results?

### Twins experiment

#### Numerical examples

<table>
<thead>
<tr>
<th>Input: $\tilde{\theta} = [0, 0, 0, 0]$ (No information)</th>
<th>Structural Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One-way</strong></td>
<td><strong>Estimates</strong></td>
</tr>
<tr>
<td><strong>TRUE</strong></td>
<td><strong>Estimates</strong></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-0.1</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-2</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>-1.5</td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>-4</td>
</tr>
<tr>
<td><strong>total error</strong></td>
<td>3.643</td>
</tr>
</tbody>
</table>

| sample | 350 | 350 |
| L0 | -373.221 | -371.887 |
| LL | -269.872 | -211.308 |
| $\rho^2$ | 0.266 | 0.421 |
| iteration | 6 | |

<table>
<thead>
<tr>
<th>Input: $\tilde{\theta} = [-1.5, -0.1, -2, -10]$ (Wrong values)</th>
<th>Structural Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One-way</strong></td>
<td><strong>Estimates</strong></td>
</tr>
<tr>
<td><strong>TRUE</strong></td>
<td><strong>Estimates</strong></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-0.1</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-2</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>-1.5</td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>-4</td>
</tr>
<tr>
<td><strong>total error</strong></td>
<td>6.058</td>
</tr>
</tbody>
</table>

| sample | 350 | 350 |
| L0 | -373.560 | -371.887 |
| LL | -328.587 | -211.308 |
| $\rho^2$ | 0.110 | 0.421 |
| iteration | 8 | |
Real data

*Matsuyama Probe Person data in 2007, 30 pedestrians, 729 locations*
Real data | Model specification

- **Route choice model**: (static) Markov model
- **Target**: Pedestrian trip in city center

- Utility function:

\[ v(a \mid k) = \theta_1 TT_a + \theta_2 CU_a + \theta_3 DU_a + \theta_4 UT_{al} \]

- \( TT \): Travel time (min.)
- \( CU \): Sidewalk width (m)
- \( DU \): Arcade dummy variable
- \( UT \): U-turn dummy variable
Real data | Parameter estimation results

- Travel time ($\theta_1$) seems to be significant from the result of one-way model, however,
- **Structural estimation** results show that links with arcade ($\theta_3$) are the most likely to be passed by pedestrians; travel time ($\theta_1$) is not significant
- Other t-values and rho-square ($\rho^2$) indicate that the **structural estimation** improves parameter estimation results

<table>
<thead>
<tr>
<th>Input: $\tilde{\theta} = [0, 0, 0, 0]$ (No information)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>One-way</td>
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</tr>
<tr>
<td>$\theta_3$</td>
</tr>
<tr>
<td>$\theta_4$</td>
</tr>
<tr>
<td>sample</td>
</tr>
<tr>
<td>L0</td>
</tr>
<tr>
<td>LL</td>
</tr>
<tr>
<td>$\rho^2$</td>
</tr>
<tr>
<td>iteration</td>
</tr>
</tbody>
</table>
Real data | Estimation results of error variance

Estimated measurement error variance is dependent on each link

Real data
Numerical examples

Estimated measurement error variance is dependent on each link

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_a$</td>
<td>31.622</td>
<td>941.021</td>
</tr>
</tbody>
</table>

Estimation result of $\sigma$

- ~ 20 (or no measurement)
- 20 ~ 30
- 30 ~ 50
- 50 ~ 100
- 100 ~
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Conclusions and Future work

• Conclusions
  – A link-based measurement model with route choice model
  – Estimation of measurement parameter for each link
  – Structural estimation method for solving a fixed point problem regarding route choice parameters

• Future work
  – Comparison of computational efficiency with previous measurement models
  – Alternatives and utility of pedestrian link choice
  – Characteristics of the fixed point problem
Thank you for attention!

Questions?

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References

Appendix
Previous route measurement models

Geometric
White et al. (2000)
- Point-to-point
- Point-to-curve
- Curve-to-curve

Topological
Greenfield (2002)
- Adjacency
- Connectivity
- Vehicle heading

Probabilistic
Ouchieng et al. (2004)
Quddus et al. (2006)
- Error region
- Fuzzy logic

Path-based
Pyo et al. (2006)
Bielraire et al. (2013)
- MHT
- Measurement equation
Link switching | errors

Difficulties regarding link connectivity because of myopic optimization
**Link switching | algorithm**

**STEP1:** Calculating probabilities

\[
p_i(a_t \mid \hat{m}_t, a_{t-1}) = \frac{p(\hat{m}_i \mid a_i; \sigma_{a_i}) p(a_i \mid a_{t-1}; \theta)}{\sum_{a_i \in A(a_{t-1})} p(\hat{m}_i \mid a_i; \sigma_{a_i}) p(a_i \mid a_{t-1}; \theta)}
\]

**STEP2:** Sorting candidates by probabilities

\[
[a_{t,1}, \ldots, a_{t,r}, \ldots, a_{t,|A(a_{t-1})|}]
\]

\[
p(a_{t,1}) \geq \cdots \geq p(a_{t,r}) \geq \cdots \geq p(a_{t,|A(a_{t-1})|})
\]

\[
r = 1
\]

**STEP3:** Calculating measurement equation at \((t+1)\)

\[
LLm_r = \log \left( p(\hat{m}_{t+1} \mid a_{t+1}; \sigma_{a_{t+1}}) \right)
\]

where, \(a_{t+1} = \arg\max_a p(a \mid \hat{m}_{t+1}, a_{t,r})\)

\[
\frac{LLm_r}{J} < \gamma
\]

\[
\text{No} \quad \text{Yes} \quad \text{No} \quad \text{Yes}
\]

\[
r = |A(a_{t-1})| \quad a_t = a_{t,r}
\]

\[
r := r+1 \quad r := 1
\]

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Twins experiments | iterations

Real data  Numerical examples

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