Smart transit systems for even smarter travellers

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Overview

• Part 1: Single line, bus bunching

• Part 2a: Transit Route choice as game
• Part 2b: Notes on extension to all choices from O to D

• Conclusions/ current work
Bus bunching

Without delay

Initial delay for 2nd bus at stop 0
Bus bunching
Likely passenger stop arrival patterns with RTI

\[ q(t) \times 10^{-3} \quad \text{passenger arrival time probability} \]
If the boarding rate is low, the service can be severely disrupted even without exogenous delays.
Influence of boarding rate on bunching

queue processes
basic scenario with b=11.11, \( p=0.15 \); \( Q=100 \)
scheduled departures @ 8.5, 18.5

queue processes
basic scenario with b=2.78, \( p=0.6 \); \( Q=100 \)
scheduled departures @ 4, 14, 24
Conclusion:

• RTI “disturbs” passenger arrival patterns

• This means the system can be much easier perturbed - Even without exogenous delay

• Holding strategies become even more important
p.s. on control strategies

- All holding strategies introduce delays
- One control strategy is “the unfriendly bus driver”: *Go to the back bus, I am leaving.*
Part 2: Route choice in transit networks

Choosing a hyperpath consists of two steps

- **Defining a set of paths**
- **Defining the selection criteria of a specific path**
Spiess and Florian “Optimal strategies”

- Spiess and Florian (1989) proposed that passengers board the first line among a set of attractive lines at a boarding node $i$.
- Finding the optimal path set can be presented as a linear program where the objective is to find the strategy that minimises the *expected* waiting time.

$$p_a(A_i^+) = \frac{f_a}{\sum_{a \in A_i^+} f_a} \quad w(A_i^+) = \frac{\alpha}{\sum_{a \in A_i^+} f_a}$$
Games and route choice

- “Hyperpaths” have also been applied in other contexts
- Risk-averse assignment leads to the creation of a set of paths in order to minimise the maximum travel cost
- Route choice as a game against single or multiple “demons” have been introduced to find worst case scenarios.
  - Bell (2000): Router vs. demon to find critical links
  - Cassir and Bell (2000): Extension to multiple travellers
  - Cassir et al (2003): Tree spoiler to investigate reliability of specific Ods
  - Szeto et al (2007): Extension to multiple (independent) demons
Proposition: \( S\&F = \text{game} \)

• The risk-averse traveller, who *fears* a maximum delay of \( d_a = 1/f_a \) on any link should use the S &F path split probabilities, independent of the travel time on any downstream link \( c_a \), and include all links that are not dominated by any other link.

• Note the difference in interpretation:
  – S&F : \( 1/f_a \) is the expected waiting time
  – Here : \( 1/f_a \) is the maximum link delay
The risk averse traveller fears that a line might be delayed by up to $d_a$, this can be described as a game with following pay-off matrix:

<table>
<thead>
<tr>
<th></th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>...</th>
<th>$q_k$</th>
<th>...</th>
<th>$q_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>$u_1 + d_1$</td>
<td>$u_1$</td>
<td>...</td>
<td>$u_1$</td>
<td>...</td>
<td>$u_1$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$u_2$</td>
<td>$u_2 + d_2$</td>
<td>...</td>
<td>$u_2$</td>
<td>...</td>
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<td>...</td>
</tr>
<tr>
<td>$p_k$</td>
<td>$u_k$</td>
<td>$u_k$</td>
<td>...</td>
<td>$u_k + d_k$</td>
<td>...</td>
<td>$u_k$</td>
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<td>...</td>
</tr>
<tr>
<td>$p_n$</td>
<td>$u_n$</td>
<td>$u_n$</td>
<td>...</td>
<td>$u_n$</td>
<td>...</td>
<td>$u_n + d_n$</td>
</tr>
</tbody>
</table>

..leading to following optimisation problem:

\[
\min_p \ \max_q \ \sum_{a \in A_i^+} u_a p_a + q_a p_a d_a
\]
Proof (2)

- The travellers is hence to choose a (mixed) strategy \( p \) that minimises his feared cost of travel \( \lambda_i \).

Min \( \lambda_i \) so that

\[
\begin{align*}
  p_1(u_1 + d_1) + p_2u_2 + \ldots + p_ku_k &= g_1 \leq \lambda_i \\
p_1u_1 + p_2(u_2+d_2) + \ldots + p_ku_k &= g_2 \leq \lambda_i \\
\vdots + \ldots + \ldots + \ldots + \ldots &= \ldots \leq \lambda_i \\
p_1u_1 + p_2u_2 + \ldots + p_k(u_k+d_k) &= g_k \leq \lambda_i \\
p_1 + p_2 + \ldots + p_k &= 1 \\
p_i > 0 \quad \forall i = 1, \ldots, k
\end{align*}
\]

- Following the expected value principle at the saddle point the costs of all used strategies will be equal.
Proof of proposition 1 (3)

• ...hence solving the set of equations wlog for $p_1$ leads to

$$p_a = p_1 \frac{d_1}{d_a} \quad \forall a=2,\ldots,k$$

and

$$p_1 + p_1 \frac{d_1}{d_2} + p_1 \frac{d_1}{d_3} + \ldots + p_1 \frac{d_1}{d_k} = 1$$

• Solving for $p_1$ leads to:

$$p_1 = \frac{\frac{1}{d_1}}{\sum_{a=1,\ldots,k} \frac{1}{d_a}}$$

qed
Further properties of this zero-sum game

• With $p_a$ determined it follows for the expected game value:

$$g = \left( u_1 + \frac{1}{f_1} \right) \frac{f_1}{\sum_i f_i} + u_2 \frac{f_2}{\sum_i f_i} + \ldots + s_n \frac{f_n}{\sum_i f_i} = \frac{1 + \sum_i f_i u_i}{\sum_i f_i}$$

• which is also equivalent to the S&F solution.

• In the same way as for the path split probabilities the attack probabilities $q_a$ can be found for the Nash equilibrium solution.
Equivalence of S&F linear program with “Multiple local demon game”

• Spiess and Florian showed that following LP determines the optimal hyperpath (with assumptions as before)

\[ \begin{align*}
\text{Min}_{p,w} & \sum_{a \in A} c_a p_a + \sum_{i \in I} w_i \\
\text{Subject to} & \sum_{a \in A_i^+} p_a - \sum_{a \in A_i^-} p_a = g_i \\
& p_a d_a \leq w_i \\
& p_a \geq 0
\end{align*} \]

• The corresponding Lagrangian function for this LP is:

\[ L(p, w, \lambda, q) = \sum_{a \in A} c_a p_a + \sum_{i \in I} w_i - \sum_{i \in I} \sum_{a \in A_i^+} q_a (w_i - p_a d_a) + \sum_{i \in I} \lambda_i (g_i - \sum_{a \in A_i^+} p_a + \sum_{a \in A_i^-} p_a) \]
Equivalence of S&F linear program with “Multiple local demon game” (2)

\[ L(p, w, \lambda, q) = \sum_{a \in A} c_a p_a + \sum_{i \in I} w_i - \sum_{i \in I} \sum_{a \in A_i^+} q_a (w_i - p_a d_a) + \sum_{i \in I} \lambda_i (g_i - \sum_{a \in A_i^+} p_a + \sum_{a \in A_i^-} p_a) \]

- The primary ↔ dual variables are \( p \leftrightarrow \lambda \) and \( w \leftrightarrow q \)
  - Ahuja et al (1993) show that the dual variable of link choice probabilities \( p \) can be interpreted as node potential (as in Proposition 1)
  - Interpretation of \( q \) ...to follow
- \( L(p,w,\lambda,q) \) is
  - Minimised with regards to \( p \) and \( w \)
  - Maxmised with regards to \( q \) and \( \lambda \)
  - Subject to non-negativity conditions on \( p \) and \( q \)
Equivalence of S&F linear program with “Multiple local demon game” (3)

• The interpretation of \( q \) is clarified by the dual problem

• \( L(p, w, \lambda, q) \) can be transformed into

\[
L(p, w, \lambda, q) = \sum_{i \in I} \lambda_i g_i - \sum_{a=(i, j) \in A} p_a (\lambda_i - \lambda_j - c_a - q_a d_a) \\
- \sum_{i \in I} w_i (\sum_{a \in A_i^+} q_a - 1)
\]

• Meaning that the dual problem can be formulated as

\[
\text{Max} \sum_{i \in I} \lambda_i g_i \\
\text{Subject to} \\
(\lambda_i - \lambda_j - q_a d_a) \leq c_a \\
\sum_{a \in A_i^+} q_a - 1 = 0 \\
q_a \geq 0
\]

Demon Problem: Maximise node costs subject to \( q_i = 1 \) at each node
Equivalence of S&F linear program with “Multiple local demon game” (4)

- Further, \( L(p, w, \lambda, q) \) can be transformed into

\[
\begin{align*}
\max_q \min_p & \sum_{a \in A} (c_a p_a + q_a d_a p_a) \\
\text{Subject to} & \sum_{a \in A^+} p_a - \sum_{a \in A^-} p_a = g_i \\
& 1 = \sum_{a \in A^+} q_a \\
& p_a \geq 0 \\
& q_a \geq 0
\end{align*}
\]

- This MaxMin problem gives the mixed strategy Nash equilibrium of a zero sum, non-cooperative game between a network user endeavouring to minimise his travel cost and node specific demons aiming to penalise the traveller by imposing delays.
Comparison to single demon game

- As pointed out the single demon game has been of special interest in the network reliability literature.
- The S&F / Multiple local demon game can be transformed into the single demon game by a change in the demon constrained:

\[
\begin{align*}
\text{Max}_q \text{Min}_p & \sum_{a \in A} (c_a p_a + q_a d_a p_a) \\
\text{Subject to} & \sum_{a \in A_i^+} p_a - \sum_{a \in A_i^-} p_a = g_i \\
& 1 = \sum_{a \in A} q_a \\
& p_a \geq 0 \\
& q_a \geq 0
\end{align*}
\]
Observation (can be proven)

- **Only the single demon game necessarily includes the shortest path**
  - In the multiple local demon game the traveller might avoid nodes with potentially large delays altogether.
  - In the single demon game the total resource of the demon is limited, meaning that the traveller should include (at least with a small probability) every potentially shortest path.
Numerical Example

- Shortest undelayed path $L_2(A-B) \rightarrow L_3(B-C-D)$ only included in single demon game

<table>
<thead>
<tr>
<th>Link usage</th>
<th>Multiple local demons</th>
<th>Single demon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_a$</td>
<td>$q_a$</td>
</tr>
<tr>
<td>L1</td>
<td>0.5</td>
<td>0.46</td>
</tr>
<tr>
<td>L2 (A-B)</td>
<td>0.5</td>
<td>0.54</td>
</tr>
<tr>
<td>L2(B-C)</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>L3(B-C)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>L3(C-D)</td>
<td>0.08</td>
<td>0.5</td>
</tr>
<tr>
<td>L4</td>
<td>0.42</td>
<td>0.5</td>
</tr>
<tr>
<td>Game value/feared travel cost</td>
<td>27.75 min</td>
<td>23.40 min</td>
</tr>
</tbody>
</table>
Numerical Example (2)

- Only \( q \) changes in MLDG
- L1 now also included in SDG

<table>
<thead>
<tr>
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<th>Multiple local demons</th>
<th>Single demon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p_a )</td>
<td>( q_a )</td>
</tr>
<tr>
<td>L1</td>
<td>0.50</td>
<td>0.63</td>
</tr>
<tr>
<td>L2 (A-B)</td>
<td>0.50</td>
<td>0.37</td>
</tr>
<tr>
<td>L2(B-C)</td>
<td>0.50</td>
<td>0</td>
</tr>
<tr>
<td>L3(B-C)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>L3(C-D)</td>
<td>0.08</td>
<td>0.50</td>
</tr>
<tr>
<td>L4</td>
<td>0.42</td>
<td>0.50</td>
</tr>
<tr>
<td>Game value/feared travel cost</td>
<td>26.75 min</td>
<td>23.20 min</td>
</tr>
</tbody>
</table>
Numerical Example (3)

- Only single route in MLDG
- Multiple routes in SDG (as 24min > travel time on shortest path)

<table>
<thead>
<tr>
<th>Link usage</th>
<th>Multiple local demons</th>
<th>Single demon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_a$</td>
<td>$q_a$</td>
</tr>
<tr>
<td>L1</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>L2 (A-B)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L2(B-C)</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>L3(B-C)</td>
<td>0</td>
<td>0.67</td>
</tr>
<tr>
<td>L3(C-D)</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>L4</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Game value/feared travel cost</td>
<td>24.00 min</td>
<td>23.06 min</td>
</tr>
</tbody>
</table>
Conclusions

• The S&F hyperpath concept can be interpreted as a “multiple local demon game” (MLDG).
  – It is the route choice of an intelligent traveller fearing that “something can go wrong at each decision point”.
  – The dual variable \( q \) is an indicator for the link importance

• A smart transit system with RTI and line coordination can reduce the travellers’ game to a single demon game.
## Extension: Effects of information on trip stages

<table>
<thead>
<tr>
<th>Travel stage</th>
<th>Uninformed, unfamiliar traveller</th>
<th>Commuter; Local dynamic information</th>
<th>Ubiquitous Info (Smart phone)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Before departure</strong></td>
<td>Look up timetable, select stop and dep. time</td>
<td>Select stop and dep. time, possibly complex alternatives and more complex strategies</td>
<td>-</td>
</tr>
<tr>
<td><strong>At stop</strong></td>
<td>Wait for service</td>
<td>possibly consider more connections</td>
<td>+ possibly consider changing stop</td>
</tr>
<tr>
<td><strong>On-Board</strong></td>
<td>No decision</td>
<td>possibly adjust alighting point</td>
<td>+ possibly change dest.</td>
</tr>
<tr>
<td><strong>At Destination</strong></td>
<td>Stick to plan or make an effort to obtain new information</td>
<td>-</td>
<td>Possibly revise and co-ordinate plans; e.g. adjust pick-up</td>
</tr>
</tbody>
</table>
Network Example
Network Example

Without hyperpath consideration
Network Example

With platform hyperpath consideration
Network Example

With **Route + Platform Hyperpath** (ITS: Information at Station entrance)
Network Example

With Walking, Route and Platform Hyperpath (Smart Phone)
Note on Hyperpath vs Strategy

- Through reliable, “dynamic” information on service departures, route choice decision points move closer to passenger departure time: At decision point the hyperpath collapses into a single path.

- Therefore for the passenger the hyperpath might not necessarily become more complex, but only “from a modelling perspective”.

Overall Conclusions

• “Smart transit”, i.e. providing good information allows the traveller to benefit by less need to be risk-averse and by having wider options
• This might in turn help the system

• But there is also a danger that the traveller “outsmarts” the system
  – Headway perturbations
  – Focus on the shortest route
• ...requiring possibly even more information and network management
Current work connected to the topic...

- Fare structures: Increasingly complex to manage and attract demand
  - Zones vs distance based and flat fares
  - Special discounts: OD pairs, peak times, loyalty rewards (per day, per month..), Shopping points
- Bus bunching and stop layout: in how far can “intelligent design” make a system robust against delays
Thank you

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